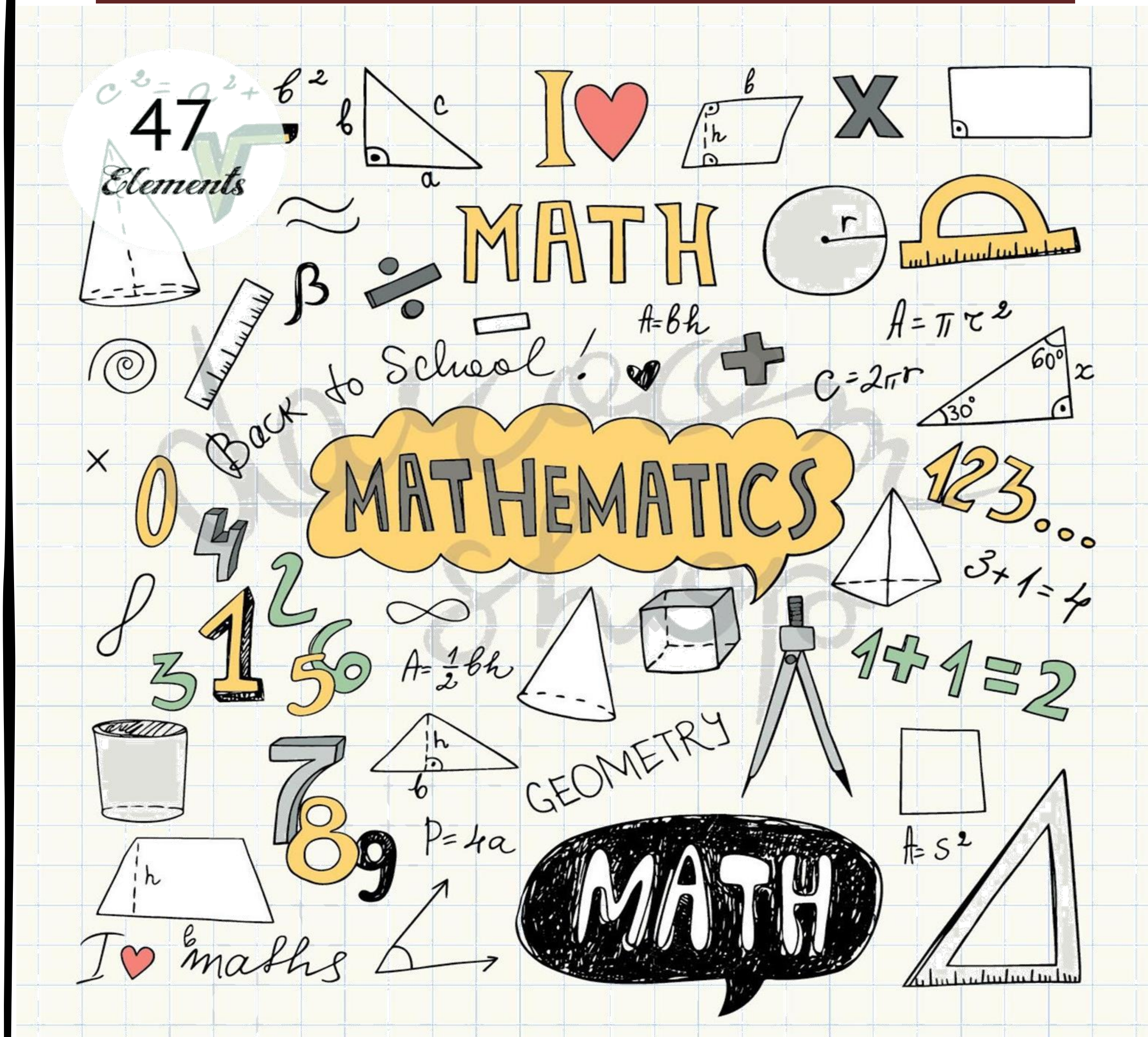
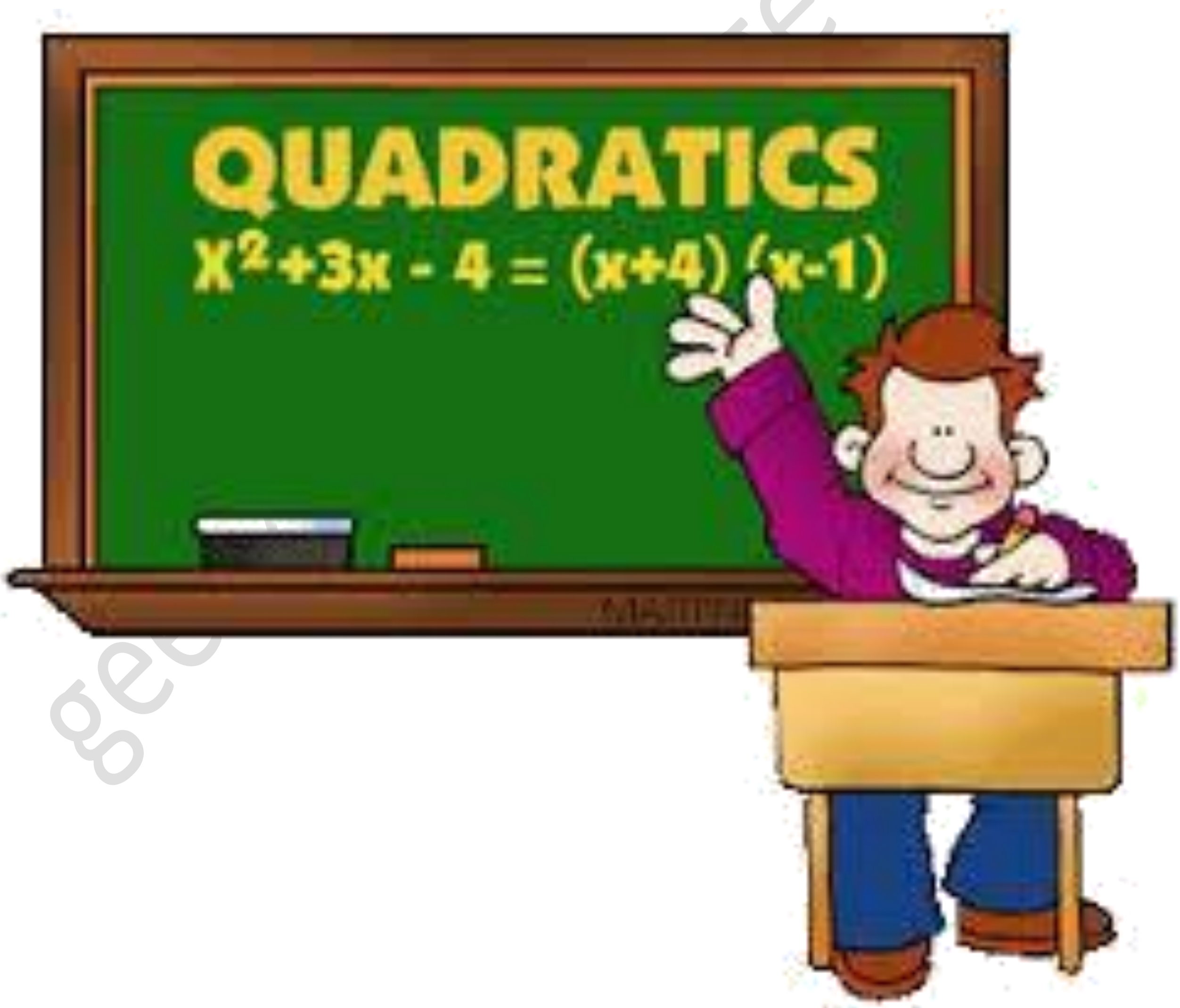


FIRST SECONDARY

FIRST TERM

Name : ………………………………


ALGEBRA


QUADRATICS
X²+3x - 4 = (x+4) (x-1)

Lesson (1)
Solving quadratic equations in

First: **Solving the quadratic equations in one variable algebraically:**

(1) By factorization

(2) by the general formula

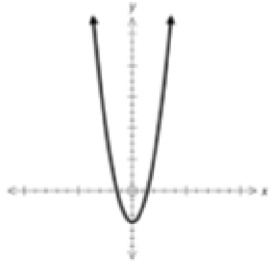
$$x = \frac{\dots\dots\dots\dots}{2a}$$

Where **a** is coefficient of x^2, **b** is coefficient of x and **c** is the absolute term.

Second: Solving the quadratic equation in one variable graphically:

Case (1)

The curve intersects x – axis at two points

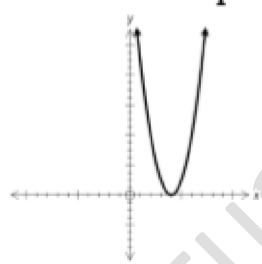


There are two solutions in R

The S.S.={L , M}

Case (2)

The curve touches x – axis at one point

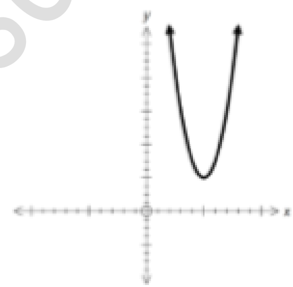


There is a unique solution in R

The S.S.={L}

Case (3)

The curve does not intersect x – axis



There is no solution in R

The S. S.=∅

Remark

In case of the interval is not given, then we can graph the function by finding the Vertex of the curve which is $(\frac{-b}{2a}, f(\frac{-b}{2a}))$, and then we find some points to the right of it , and the same number of points to the left of It.

Exercises (1)

1)Find Algebraically the S.S in R:

(1) $x^2 - 1 = 0$ (2) $x^2 + 9 = 0$

(3) $x^2 + 3x = 0$ (4) $x^2 - 6x + 9 = 0$

Solution

(1)…………………………………………………………………………………
(2) …………………………………………………………………………………
(3) …………………………………………………………………………………
(4) …………………………………………………………………………………

2) Find the S.S of the following equations using the general formula :

(1) $x^2 - x + 7 = 0$ << knowing that $\sqrt{2} = 1.4$>>

(2) $3x^2 - 65 = 0$ << Approximate the result to the nearest tenth >>

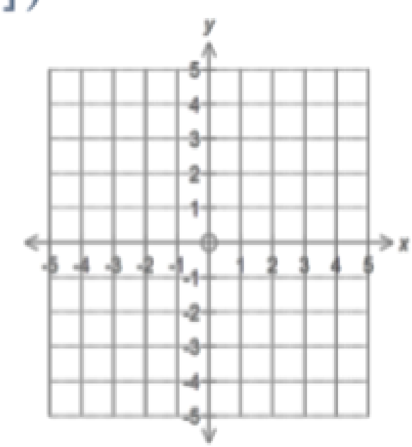
Solution

(1)…………………………………………………………………………………
…………………………………………………………………………………
(2)…………………………………………………………………………………
…………………………………………………………………………………
…………………………………………………………………………………

3) Find in R the solution set of the following equations graphically:

(1) $3x - x^2 + 2 = 0$ (Draw in the interval [- 1 , 4])

x	-1	0	1	2	3	4
F(x)						

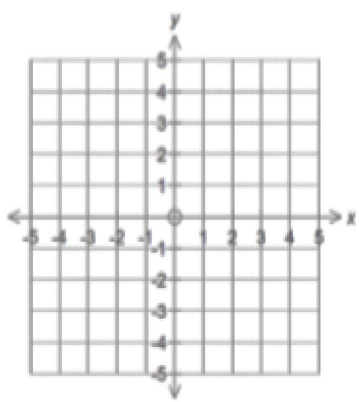


(2) $-2x^2 - 4x + 1 = 0$

$\frac{-b}{2a} = ….$

$f\left(\frac{-b}{2a}\right) = …..$

The vertex is ………



Lesson (2)

An introduction in complex number

The imaginary number " i "

The imaginary number " i " is **defined** as the number whose square is **– 1**,

$i^2 = -1$

- **We can write the square roots of a negative numbers as follows:**

$\sqrt{-2} = \sqrt{2\,i^2} = \sqrt{2}i$ ، $\sqrt{-3} = \sqrt{3\,i^2} = \sqrt{3}i$

$\sqrt{-5}$ =………………… , $\sqrt{-9}$ = …………………

$\sqrt{-6} \times \sqrt{3} =$ ………… , $\sqrt{-5} \times \sqrt{-6} =$ …………

Find the S.S. in the set of imaginary numbers:

1) $x^2 + 25 = 0$

……………………………………………………………………………………………

……………………………………………………………………………………………

2) $3\,x^2 + 27 = 0$

……………………………………………………………………………………………

……………………………………………………………………………………………

Integer powers of " i ":

$i^1 = i$	$i^2 = -1$	$i^3 = -i$	$i^4 = 1$
$i^5 = i$	$i^6 = -i$	$i^7 = -i$	$i^8 = 1$
$i^{4n+1} = i$	$i^{4n+2} = -1$	$i^{4n+3} = -i$	$i^{4n+4} = 1$

1]Find each of the following in the simplest form:

a) i^9 ……………………………… b) i^{38} ………………………………

c) i^{10} ……………………………… d) i^{4n+2} ………………………………

e) i^{-9} ……………………………… f) i^{-43} ………………………………

g) $\frac{1}{i^4} =$ ……………………………… h) $\frac{1}{i^9} =$ ………………………………

I) $\frac{1}{i^{19}} =$ ………………………………

The Complex number

The Complex number is the number that can be written in the form: $a + bi$.
Where a and b are two real numbers and $i^2 = -1$

- a is called the real part.
- bi is called the imaginary part.

Set of Complex numbers

The Set of Complex number C is defined as :
$C = \{ a + bi : a \in R, b \in R, i^2 = -1\}$

Operations on the complex numbers:

Find the result of each of the following in the simplest form:

a) $(3 + 7i) + (5 - 9i) =$

....

b) $(2 - 4i) - (3 + 5i) =$

....

c) $(4+3i)(2 - 5i) =$

....

d) $(3+2i)^2 =$

....

e) $(1 - i)^4 =$

....

f) $(1 - i)^{10} =$

....

Equality of two Complex numbers

Two Complex numbers are **equal** if and only if ($\Leftrightarrow$) the two **real** parts are **equal** and the two **imaginary** parts are **equal**.
i.e. if : (a + bi) and (c + di) are two Complex numbers and if: a = c ,b = d ,then : a +bi = c + di and vice versa: if: a + bi = c + di , then : a = c , b = d

Find the values of x and y which satisfy the equation:

a) $x + y\,i = 4 + 5\,i$

....

b) $x - 3y + (2x + y)\,i = 6 + 5\,i$

....

....

c) $4x - y + (2x + y)\,i = 5 + 7\,i$

....

....

The two Conjugate numbers

The two numbers: $a + bi$ and $a - bi$ are called Conjugate numbers.

Note: Take care that the Complex number and its Conjugate differ only in the sign of their imaginary parts.

For example:

$2 + 5\,i$ and $2 - 5\,i$ are conjugate numbers

$3\,i - 7$ and $-3i - 7$ are conjugate numbers (-7+3i) and (-7 – 3i)

1]Simplify:

a) $\frac{10}{3+i}$ = ……………………………………………………………………………………

b) $\frac{3+2i}{2-5i}$ = ……………………………………………………………………………………

2] Find the value of x and y that satisfy of the following equation :

1) $\frac{10}{2+i} = x + yi$

…………………………………………………………………………………………

2) $\frac{6-4i}{1-i} = x + yi$

…………………………………………………………………………………………

Exercises (2)

1) Simplify each of the following:

(1) $\sqrt{-36}$ (2) $\sqrt{-18} \times \sqrt{-12}$

…………………………………………………………………………………………

(3) (- 4 i) (- 6 i) (4) $(-2i)^3(-3i)^2$

…………………………………………………………………………………………

2) Find the result of each of the following in the simplest form:

(1) $(3 + 2i) + (2 - 5i)$ ……………………………………………………

(2) $(12 - 5i) - (7 - 9i)$ ……………………………………………………

(3) $(2 + 3i)(3 - 4i)$ ……………………………………………………

(4) $(4 - 3i)(4 + 3i)$ ……………………………………………………

3) put each of the following in the form (a + b i)

(1) $\frac{4+i}{i}$ … … … … … … … … … … … …

(2) $\frac{2}{1+i}$ … … … … … … … …

(3) $\frac{26}{3-2i}$ ……………………………………

(4) $\frac{2-3i}{3+i}$ … … … … … … … … … … …

(5) $\frac{3+4i}{5-2i}$ … … … … … … … … … …

(6) $\frac{(3+i)(3-i)}{3-4i}$ … … … … … … …

4) Solve each of the following equations:

(1) $3x^2 + 12 = 0$ …

(2) $\frac{3}{5}y^2 + 15 = 0$ …

(3) $4x^2 + 100 = 75$ …

Lesson (3)
Determining the type of roots of a quadratic equation

Discriminate

***The expression**: $b^2 - 4ac$ is called the discriminant of the quadratic equation Because it is used to determine the types of roots of the quadratic equation as follows:

Discriminant	The types of the two roots	A sketch for the function related to the equation
Is positive $(b^2 - 4ac) > 0$	Two different real roots	
Is equal to zero $b^2 - 4ac = 0$	Two equal real roots	
is negative $b^2 - 4ac < 0$	Two Complex and non Real roots	

Determine the type of roots without solving:

1) $x^2 - 7x + 10 = 0$

…………………………………………………………………………………………

2) $x^2 + 4x + 5 = 0$

…………………………………………………………………………………………

3) $4x^2 - 12x = -9$

…………………………………………………………………………………………

(1) If the coefficient a , b and c in the quadratic equation: $ax^2 + bx + c = 0$ are rational numbers and the discriminate is a perfect square ,then the roots are real rational numbers.

(2) If the discriminant of the quadratic equation isn't positive, then the two roots of the quadratic equation are complex numbers and conjugate.

1]Find the value of K in each of the following cases:

a) If the two roots of the equation : $x^2 + 4x + k = 0$ are real and different .

....

....

b) If the two roots of the equation : $5x^2 + 4x + k = 0$are Complex and not real.

....

....

c) If the equation : $x^2 = k + 2$ has two real different roots.

....

....

2] IF a and b are rational no. prove that the two roots of the equation $ax^2 + bx + b - a = 0$ are rational.

....

....

....

Exercises (3)

1) Determine the type of the two roots of each of the following equations :

(1) $x^2 - 3x + 4 = 0$ (2) $x^2 - 10x + 25 = 0$ (3) $3x^2 + 10x - 4 = 0$

………………………………………………………………………………………………

(4) $x(x - 2) = 5$ (5) $(x - 11) - x(x - 6) = 0$

………………………………………………………………………………………………

(6) $2x + 1 = \frac{5}{x - 3}, x \neq 3$ (7) $\frac{x}{x + 1} + \frac{x}{x - 1} = 3$

………………………………………………………………………………………………

2) Prove that: the two roots of the equations $2x^2 - 3x + 2 = 0$ are complex and not real , **then** use the general formula to find those two roots.

………………………………………………………………………………………………

………………………………………………………………………………………………

3) Find the value of k in each of the following cases:

1) If the two roots of the equation: $5x^2 + 4x + k = 0$ are real and different.

………………………………………………………………………………………………

………………………………………………………………………………………………

2) If the two roots of the equation: $kx^2 - 8x + 16 = 0$ are complex and not real

………………………………………………………………………………………………

………………………………………………………………………………………………

4) find the values of the real numbers m that satisfy the equation:

$(m - 1) x^2 - 2m x + m = 0$ has no real roots.

………………………………………………………………………………………………

………………………………………………………………………………………………

Lesson (4)
Relation between the two roots of the second degree equation and the coefficient of its terms

We know that the two roots of the quadratic equation: $ax^2 + bx + c = 0$ are:

$\frac{-b+\sqrt{b^2-4ac}}{2a}$, $\frac{-b-\sqrt{b^2-4ac}}{2a}$ Let one of the two roots be L and the other is M then:

1) $L + M = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-2b}{2a} = \frac{-b}{a}$

$$\text{The sum of the two roots} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} \Leftrightarrow L + M = \frac{-b}{a}$$

2) $LM = \frac{-b+\sqrt{b^2-4ac}}{2a} \times \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{b^2-(b^2-4ac)}{4a^2} = \frac{b^2-b^2+4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$

$$\text{The product of the two roots} = \frac{\text{Absolute term}}{\text{Coefficient of } x^2} \Leftrightarrow LM = \frac{c}{a}$$

Remarks

In the quadratic equation :$ax^2 + bx + c = 0$

1) If :$\boxed{a = 1}$, Then :L+M = -b and LM = c

2) If :$\boxed{b = 0}$, Then L+M = 0 , i.e. L = -M

i.e. one of the two roots of the equation is the additive inverse of the other.

3) If : $\boxed{a = c}$, Then :LM=1 , i.e. $L = \frac{1}{M}$

i.e. One of the two roots of the equation is the multiplicative inverse of the other.

4) If one of the two roots of the equation : $ax^2 + bx + c = 0$ is

Double the additive inverse of the other root $\Rightarrow \boxed{2b^2 + ac = 0}$

Exercises (4)

1) Without solving the equation **find** the sum and the product of the two roots of the following equations :

(1) $4x^2 + 4x - 35 = 0$

……………………………………………………………………………………………

(2) $3x^2 = 23x - 30$

……………………………………………………………………………………………

(3) $(2x - 3)(x + 2) = 0$

……………………………………………………………………………………………

---

2) If the product of the two roots of the equation: $3x^2 + 10x - c = 0$ is $\frac{-8}{3}$.
Find the value of c, and then **solve** the equation in the set of complex number.

……………………………………………………………………………………………

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3) If the sum of two roots of the equation: $2x^2 + bx - 5 = 0$ is $\frac{-3}{2}$, find the value of b , then **solve** the equation in the set of the complex number.

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4) **Find** the other root of the equation, then find the value of a in each of the following :

(a) If : x = -2 is one of the two roots of the equation: $x^2 - 2x + a = 0$

……………………………………………………………………………………………

(b) If :(1+i)is one of the two roots of the equation: $x^2 - 2x + a = 0$

……………………………………………………………………………………………

---

5) **Find** the value of a , b in each of the following equations , If:

(a) 5 , 3 are the two roots of the equation: $x^2 + ax + b = 0$

……………………………………………………………………………………………

(b) 3i , −3i are the two roots of the equation: $x^2 + ax + b = 0$

……………………………………………………………………………………………

Lesson 5

Forming the quadratic equation whose two roots are known

Let L and M be the two roots of the quadratic equation: $a\,x^2 + b\,x + c = 0$

By multiplying the two sides by $\frac{1}{a}$ where $a \neq 0$, the equation becomes in the form : $x^2 + \frac{b}{a}\,x + \frac{c}{a} = 0$ i. e. $x^2 - \left(\frac{-b}{a}\right)x + \frac{c}{a} = 0$ (1)

But $L + M = \frac{-b}{a}$, $LM = \frac{c}{a}$ By substituting in (1) , we get the quadratic equation whose roots are L,M which is: $x^2-(L+M)x+LM=0$ (2)

i.e. x^2 – (the sum of the two roots) x + product of the two roots = 0

by factorizing , we get another form of the last equation : $(x - L)(x - M)=0$

Remember the following identities:

1) $L^2 + M^2 = (L + M)^2 - 2LM$
2) $(L - M)^2 = (L + M)^2 - 4LM$
3) $L^3 + M^3 = (L + M)[(L + M)^2 - 3LM]$
4) $L^3 - M^3 = (L - M)[(L + M)^2 - LM]$
5) $\frac{1}{M} + \frac{1}{L} = \frac{L+M}{LM}$
6) $\frac{L}{M} + \frac{M}{L} + \frac{L^2+M^2}{LM} = \frac{(L+M)^2-2LM}{LM}$

Exercises (5)

1) Form the quadratic equation whose two roots are :

(1) $-2, 4$ …………………………………………………………………………

(2) $-5i, 5i$ ………………………………………………………………………

(3) $3-2\sqrt{2}\,i\,, 3+2\sqrt{2}\,i$ ………………………………………………………

2) If L and M are the two roots of the equation: $x^2-7x+5=0$ Then **Find** the numerical value of each of the following expressions

(1) L^2M+M^2L

……………………………………………………………………………………………

(2) $\frac{1}{M}+\frac{1}{L}$

……………………………………………………………………………………………

(3) $(L-2)(M-2)$

……………………………………………………………………………………………

3) If L and M are the two roots of the equation: $x^2-3x-5=0$, Then **Find** the equation whose roots are: $L-4$ and $M-4$

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

4) If L and M are the two roots of the equation : $x^2+3x-5=0$, Then **Form** the quadratic equation whose roots are : L^2 and M^2

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

5) Find the quadratic equation in which each of the two roots exceeds One of the two roots of the equation : $x^2-7x-9=0$

……………………………………………………………………………………………

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Lesson 6

Sign of a function

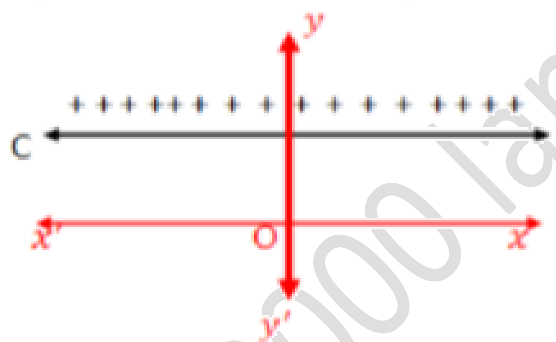
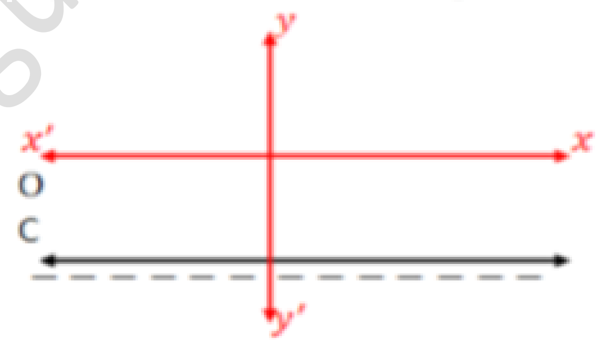
Investigating the sign of a function

Investigating the sign of a function is to determine the values of x at which The values of the function f are as follows:

Positive, i.e. f(x)>0	Negative , i.e. f(x)<0	equal to zero , i.e. f(x)=

First : The sign of the constant function

- The following two figures represent the two functions :

$f: f(x) = c$ (where c is positive)	$f: f(x) = c$(where c is negative)
	
We notice that : The function is positive for all $x \in R$	**We notice that :** The function is negative for all $\in R$

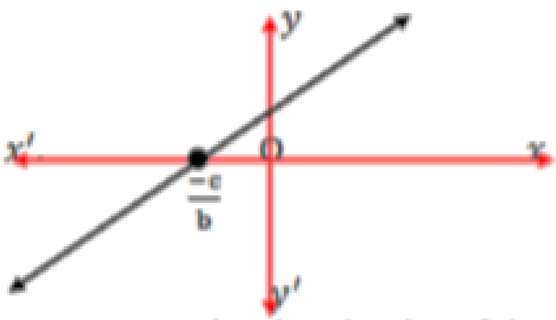
From the previous, we deduce that

The sign of the constant function $f: f(x) = c, c \in R^*$ is the same sign of c $\forall x \in R$

Second: the sign of the first degree function

The following figures represent the two functions:

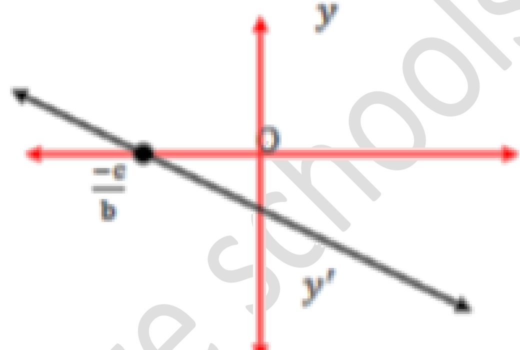
$f: f(x) = bx + c$ (b is positive)



We notice that the sign of the function :

- is the same as the sign of b(positive) at $x > \frac{-c}{b}$
- is opposite to the sign of b(negative) at $x < \frac{-c}{b}$
- equals Zero at $x = \frac{-c}{b}$

$f: f(x) = b\,x + c$ (b is negative)



We notice that the sign of the function :

- is the same as the sign of b (negative) at $x > \frac{-c}{b}$
- is opposite to the sign of b (positive) at $x < \frac{-c}{b}$
- equals Zero at $x = \frac{-c}{b}$

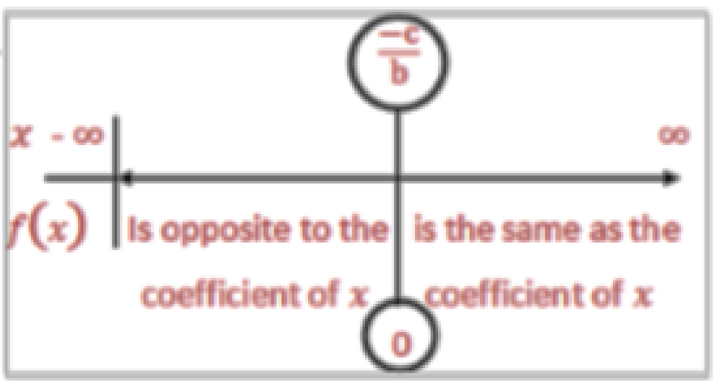
From the previous, we deduce that

To find the sign of the linear function $f: f(x) = b\,x + c,\ b \neq 0$, we put

$f(x) = 0 \therefore b\,x + c = 0 \quad \therefore x = \frac{-c}{b} \quad \therefore$ The sign of the function f :

1) Is the same as the sign of b at $x > \frac{-c}{b}$
2) Is opposite to the sign of b at $x < \frac{-c}{b}$
3) $f(x) = 0$ at $x = \frac{-c}{b}$

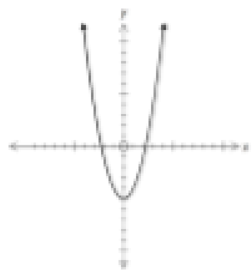
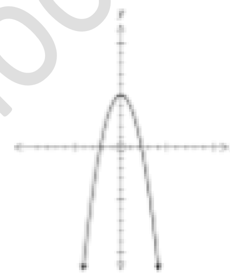
We can show that on
The number line as follows:



Third: The sign of the second degree function (quadratic function)

To determine the sign of the quadratic function f: $f(x) = a x^2 + b x + c$, $a \neq 0$
We have to obtain the discriminant of the equation: $a x^2 + b x + c =$ zero
Three cases:

1] The discriminant : $b^2 - 4ac > 0$

• If a is positive 	• If a is negative 

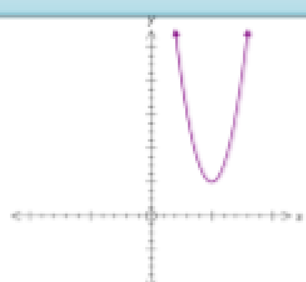
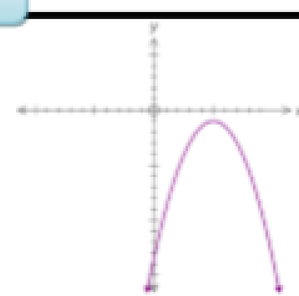
∴The equation has two roots , let them be L , M where L< M

The sign of the function is as follows :

- is the same as the sign of a at $x \in R - [L , M]$
- is opposite to the sign of a at $x \in]L , M[$
- equals zero at $x \in \{L , M\}$
- And we illustrate this on the opposite number line.

x	$-\infty$	L		M	∞
$f(x)$	the same as the sign of a	0	opposite to the sign of a	0	the same as the sign of a

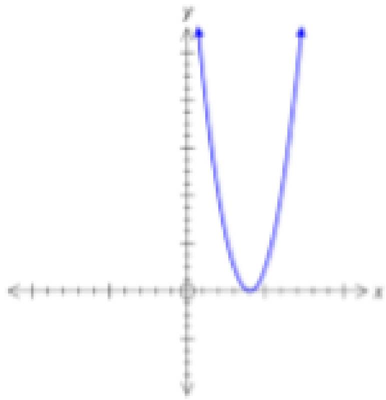
2] The discriminant : $b^2 - 4ac < 0$

If a is positive 	If a is negative 

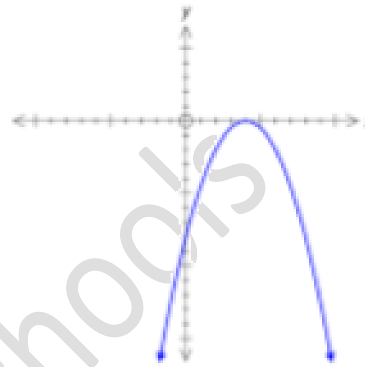
The sign of the function is the same as the sign of a $\forall \in R$

3] The discriminant : $b^2 - 4ac = 0$

If a is positive



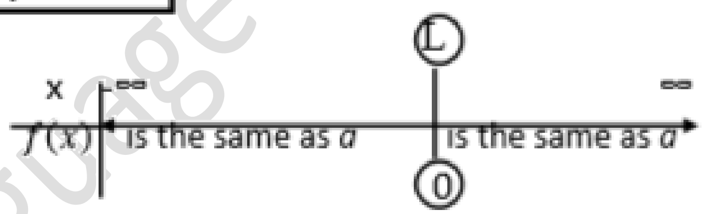
If a is negative



The sign of the function is as follows :

- Is the same as a at $x \neq L$
- Is equal to zero at x=L

We can illustrate this on the opposite number line.



1] draw the graph of the following function , then from the graph determine the sign of it:

a) $f : f(x) = x^2 - 5x + 6$ **[0,5]**

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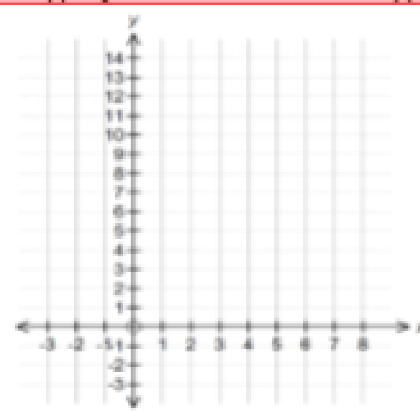

b) $f: f(x) = -x^2 + 4x - 4$ **[0,4]**

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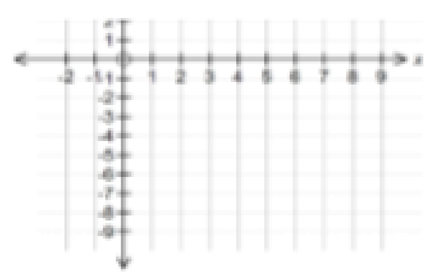

Exercises (6)

1] Investgate the sign of the functions which are defined by the following rules

(1) f(x)=-5 ……………………… (2) f(x)=2 ………………………………

(3) f(x)=2 x …………………… (4) f(x)=-3 x …………………………

(5) f(x)=2 x+4 ……………………

2]Determine the sign of each of the following functions which are defined by the following rules , Then represent your answer on the number line :

(1) $f(x) = x^2 - 8x + 16$

………………………………………………………………………………………………

(2) $f(x) = -4x^2 + 10x - 25$

………………………………………………………………………………………………

(3) $f(x) = (x - 2)(x + 3)$

………………………………………………………………………………………………

(4) $f(x) = (2x - 3)^2$

………………………………………………………………………………………………

3] **Draw** the curve of the function $f: f(x) = x^2 - 9$ in the interval [- 3 , 4].
From the graph , determine the sign of f in that interval.

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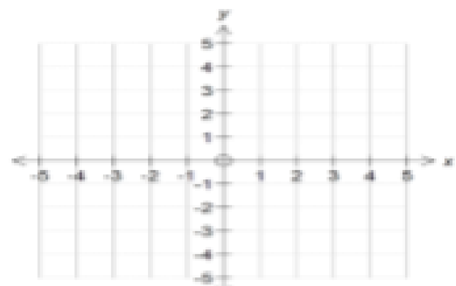

4] **Draw** the curve of the function $f: f(x) = -x^2 + 2x + 4$ in [- 3 , 5]. From the graph , determine the sign of f in that interval.

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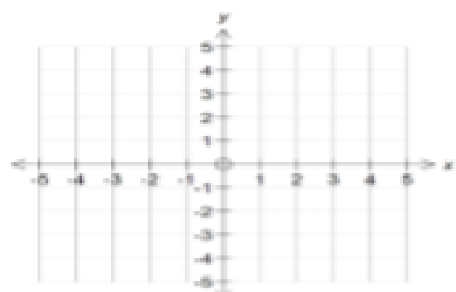

Lesson 7

Quadratic inequalities in one variable

Solving quadratic inequalities in R:

To solve the quadratic inequality, we follow the following steps:

1) We write the quadratic function related to the inequality.
2) We study the sign of the quadratic function.
3) We determine the intervals, which satisfy the inequality.

1]Example:

Find in $\mathbb{R}$ the solution set of the inequality : $x^2 - 5x + 6 > 0$

solution

2]Example:

Find in $\mathbb{R}$ the solution set of the inequality : $x^2 - 2x > 8$

solution

Exercises (7)

1] Find in R the solution set for each of the following inequalities :

(1) $x^2 + 2x - 8 > 0$

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(2) $x^2 - 1 \leq 0$

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(3) $4 - x^2 < 0$

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……………………………………………………………………………………………

(4) $x^2 - 4x + 4 \geq 0$

……………………………………………………………………………………………

……………………………………………………………………………………………

(5) $6x - x^2 - 9 < 0$

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2] Find in R the solution set of the following inequalities :

(1) $5x^2 + 12x \geq 44$

……………………………………………………………………………………………

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(2) $3x^2 \leq 11x + 4$

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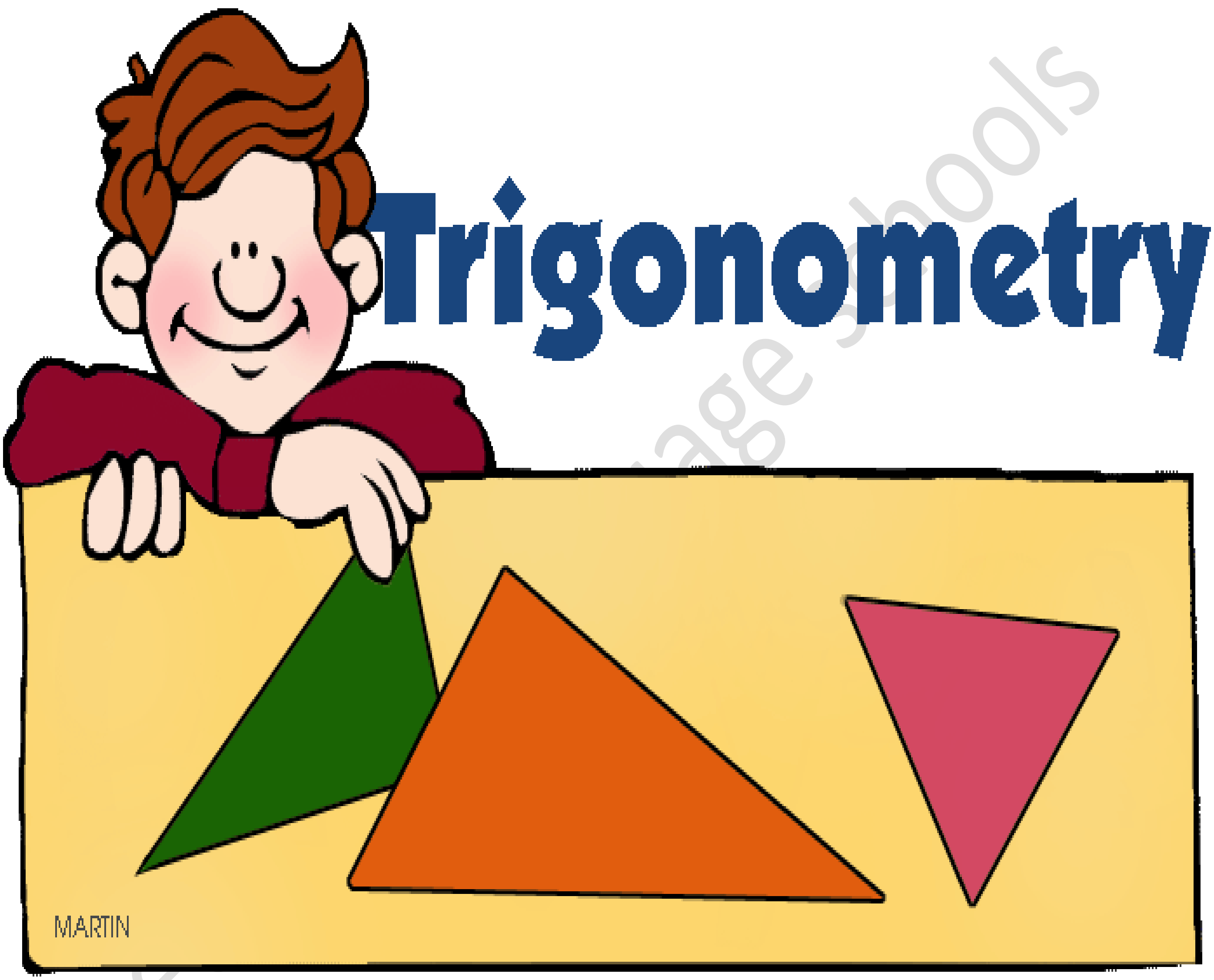

Trigonometry
MARTIN

Lesson 1

Directed angle

Directed angle

If $\overrightarrow{OA}$, $\overrightarrow{OB}$ are the two sides of an angle whose vertex is "O ", **then**:

1) $(\overrightarrow{OA}, \overrightarrow{OB})$ represents the directed angle ∠ AOB

Whose initial side $\overrightarrow{OA}$ and terminal side $\overrightarrow{OB}$

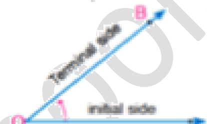


Figure (1)

2) $(\overrightarrow{OB}, \overrightarrow{OA})$ represents the directed angle ∠ BOA

Whose initial side $\overrightarrow{OB}$ and terminal side $\overrightarrow{OA}$

Figure (2)

The directed angle

It is an order pair of two rays called the sides of the angles with a common starting point called the vertex.

Remark:

$(\overrightarrow{OA}, \overrightarrow{OB}) \neq (\overrightarrow{OB}, \overrightarrow{OA})$, So: ∠AOB directed angle ≠ ∠ BOA directed angle

Positive and negative measures of a directed angle

1) The measure of the directed angle ∠AOB is positive if the direction of The rotation from the initial side to the terminal side is anti-clockwise.

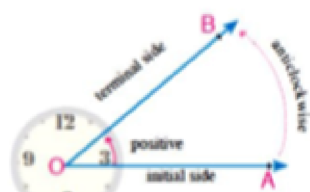


figure (1)

2) The measure of the directed angle ∠AOB is negative if the directed of the rotation from the initial side to the terminal side is clockwise.

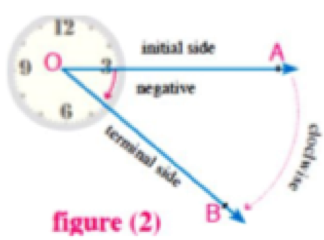


figure (2)

Remarks:

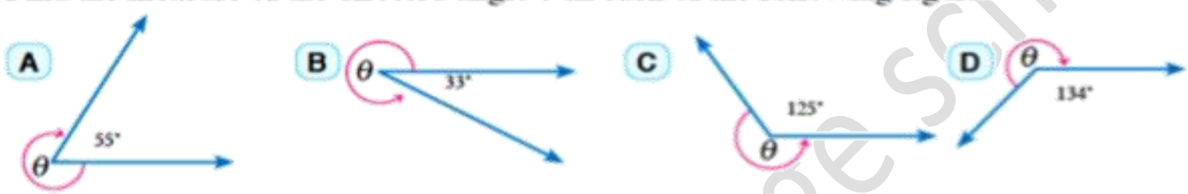
1) Each directed angle has two measures, one is positive and the other is negative such that the sum of the absolute values of the two measures = 360°.

2) The positive measure of the directed angle = θ ,

Then the negative angle = θ - 360

3) The negative measure of the directed angle = -θ , then the positive measure of the same angle = -θ +360

1 Find the measure of the directed angle θ in each of the following figure:



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Example **Complete :**

a) The +ve measure of the directed angle whose measure is (– 170°) =….

b) The –ve measure of the directed angle whose measure is (320°) = ……

c) The +ve measure of the directed angle whose measure is (– 215°) =…

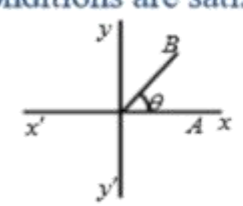
d) The –ve measure of the directed angle whose measure is (85°) =….

The Standard position of the directed angle:

A directed angle is in the standard position if the following two conditions are satisfied:

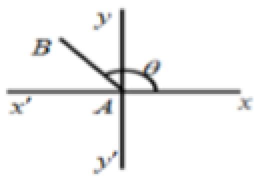
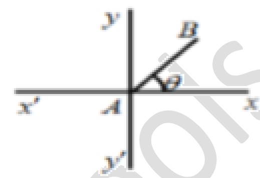
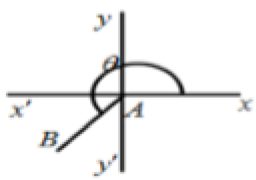
1) Its vertex is the origin point (O).

2) Its initial side lies on the positive direction of X – axis .



Angle position in the orthogonal Co- ordinate plane:

If the directed angle $\angle$ AOB is in the standard position and its positive measure θ is then its terminal side OB lies in one of the quadrants:

2^{nd} quadrant  $90^{o} < \theta < 180^{o}$	1^{st} quadrant  $0^{o} < \theta < 90^{o}$
3^{rd} quadrant  $180^{o} < \theta < 270^{o}$	4^{th} quadrant  $270^{o} < \theta < 360^{o}$

Remark:

If the terminal side lies on one of the two axes , then the angle is called **(quadrantal angle)**

i.e. The angles whose measures are 0°, 90°, 180°, 270°, 360°are quadrant angles.

Example :

Determine the quadrant in which each of the directed angles whose measures are:

213^{o} , 132^{o} , -310^{o} , -15^{o} , 270^{o}

……………………………………………………………………………………

Equivalent angles:

The directed angles are said to be equivalent if they have the same terminal side.

Example **Find a positive and a negative measure of an angle co-terminal with each of the Following angles:**

A) 120° B) – 230°

C) 285° D) 435°

…………………………………………………………………………………………

…………………………………………………………………………………………

Exercises (1)

1 Complete the following :

(1) The directed angle is ……… of two rays which are ……… with a common starting point which is ………

(2) The directed angle is in its standard position if ………

(3) The measure of the directed angle is positive if the direction of the initial side to the terminal side is ……… and negative if the direction is ………

(4) It is said that the directed angles in the standard positions are equivalent if ………

(5) If the terminal side of the directed angle lies on one of the coordinate axes , then it is called ………

Choose the right answer :

(1) The angle whose measure is 60° in the standard position is equivalent to the angl of measure ………

(a) 120° (b) 240° (c) 300° (d) 420°

(2) The angle of measure 585° is equivalent to the angle in the standard position of measure ………

(a) 45° (b) 135° (c) 225° (d) 315°

(3) The angle whose measure is 950° is equivalent to the angle in the standard position o measure ………

(a) 130° (b) – 130° (c) 235° (d) – 230°

(4) All the following angles are equivalent to 75° in the standard position except ………

(a) – 285° (b) – 645° (c) 285° (d) 435°

(5) The quadrant in which the angle of measure 1670° lies is the ………

(a) first (b) second (c) third (d) fourth

(6) The angle whose measure is (– 135°) lies in the ……… quadrant.

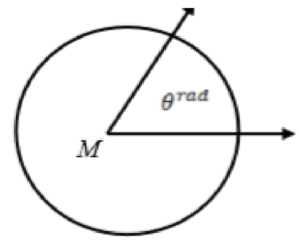
(a) first (b) second (c) third (d) fourth

Lesson 2

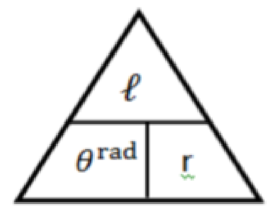
System of measuring angle

Definition

If θ^{rad} is the radian measure of the central angle in a circler l of radius length r subtends an arc of length l , Then



$$\theta^{rad} = \frac{\ell}{r}$$



$\because \theta^{rad} = \frac{\ell}{r} \qquad \therefore \ell = \theta^{rad} \times r \qquad , r = \frac{\ell}{\theta^{rad}}$

The unit of measurement of the radian angle:

It is the radian angle which is denoted by (1^{rad}) and is read as one radian.

Definition of the radian measure:

It is a central angle in a circle subtends an arc of length equals the length of the radius of the circle.

Example Find the radian measure of the central angle which subtend an arc of length l in a circle of radius r if:

a) $\ell = 15$ cm , r = 10 cm b) $\ell = \frac{2\pi}{3}$, r = 6 cm.

....

....

Example Find the length of the radius of the circle :

a) $\theta = 1.6^{rad}$, $\ell = 22.5$ cm **b)** $\theta = 2.43^{rad}$, $\ell = 43.92$ cm

....

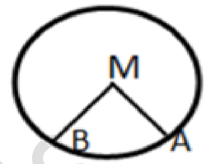
....

....

The relation between the radian measure and the degree measure:

$\because \dfrac{\text{measure of the arc}}{\text{measure of the circle}} = \dfrac{\text{length of this arc}}{\text{circumference of the circle}} \quad \therefore \dfrac{m(\widehat{AB})}{360^0} = \dfrac{\text{length of } \widehat{AB}}{2\,\pi\, r}$

$\because m(\angle AMB) = m(\widehat{AB}) \quad \therefore \dfrac{m(\angle AMB)}{180^0} = \dfrac{\text{length of } \widehat{AB}}{\pi r}$



Assuming that: m(< AMB)= x^0 in degrees , θ^{rad} in radians , length of $\widehat{AB} = \ell$

$\boxed{\therefore \dfrac{x^0}{180^0} = \dfrac{\theta^{rad}}{\pi}}$ and from it $\boxed{\theta^{rad} = x^0 \times \dfrac{\pi}{180^0}}$, $\boxed{x^0 = \theta^{rad} \times \dfrac{180^0}{\pi}}$

Example **Find in term in π the radian measure of each of the following :**

1) 135^0 2) 90^0 3) -235^0

……………………………………………………………………………………………

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Example **Find the degree measure in each of the following :**

1) $\dfrac{11\pi}{15}$ 2) 0.72π 3) 0.49^{rad} 4) -1.67^{rad}

……………………………………………………………………………………………

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Example **Complete :**

1) The angle of measure $\dfrac{25\pi}{9}$ lies in the …….. Quadrant.

2) The radian measure of the angle of measure $43^0\ 12^\backslash$ is ……..

3) The sum of measures of the quadrilateral in radian is …….

4) In a circle of diameter length 12 cm , the length of the arc subtended by a central angle of measure 60^0 = …….. cm

5) In the circle whose radius length is unit length, the measure of the central angle in radian is ………. Its length arc. (0.5 , 0.25 , 2 , 1)

Exercises (2)

Choose the right answer :

(1) The angle of measure $\frac{25\pi}{9}$ lies in the ……… quadrant.

(a) first (b) second (c) third (d) fourth

(2) The angle of measure $\frac{31\pi}{6}$ lies in the ……… quadrant.

(a) first (b) second (c) third (d) fourth

(3) The angle of measure $\frac{-9\pi}{4}$ lies in the ……… quadrant.

(a) first (b) second (c) third (d) fourth

(4) If the degree measure of an angle is $43°\ 12'$, then its radian measure is ………

(a) 0.24^{rad} (b) 0.24π (c) 0.28^{rad} (d) 0.28π

(5) The degree measure of the angle of measure $\frac{8\pi}{3}$ is ………

(a) 540° (b) 820° (c) 150° (d) 480°

(6) The sum of the measures of the angles of the quadrilateral in radian equals ………

(a) 2π (b) π (c) $\frac{3\pi}{2}$ (d) 3π

(7) If the sum of measures of the interior angles of a regular polygon equals 180° (n – 2) where n is the number of its sides , then the measure of the interior angle in radian of a regular pentagon equals ………

(a) $\frac{\pi}{3}$ (b) $\frac{7\pi}{2}$ (c) $\frac{3\pi}{5}$ (d) $\frac{2\pi}{3}$

(8) In a circle of diameter length 12 cm. , the length of the arc subtended by a central angle of measure 60° equals ……… cm.

(a) 5π (b) 4π (c) 3π (d) 2π

(9) The measure of the central angle in a circle of radius length 15 cm. and opposite to an arc of length 5π cm. equals ………

(a) 30° (b) 60° (c) 90° (d) 180°

(10) If the measure of one of the angles of a triangle is 75° and the measure of another angle is $\frac{\pi}{3}$, then the radian measure of the third angle equals ………

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{12}$

1) Find in terms of π the radian measure of each of the following angle:

a) $300°$ b) $-210°$ c) $780°$

……………………………………………………………………………………
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2) Determine the degree measure and the radian measure for the central angle that subtend an arc of length (ℓ) in a circle of radius (r) in each of the following cases:

1) $\ell = 12$cm , r = 10 cm 2) $\ell = 14$ cm , r = 7 cm

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3] Find the length of the radius of the circle in which a central angle (θ)is drawn Subtend an arc of length (ℓ)in a circle of radius(r)in each of the following Cases:

(1) $\theta = \frac{9\pi}{8}$, $\ell = 22.5$ cm. (2) $\theta = 0.767^{rad}$, $\ell = 38.35$ cm.

(3) $\theta = 139^0$, $\ell = 24.325$ cm. (4) $\theta = 78^0 36' 26''$, $\ell = 43.92$ cm.

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4]Find to the nearest one decimal place of a centimeter the length of an arc in a circle of radius length(r) subtending a central angle of measure(θ) in each of the following :

(1) r = 12.5 cm. , $\theta = 1.6^{rad}$ (2) r = 7.5 cm. , $\theta = 67^0 40'$

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5] Find the circumference of a circle which has an arc length of 12 cm. subtended by an inscribed angle of measure 45^0

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6] If the measure of a central angle in a circle equals 105^0 subtending an arc of length $\frac{7\pi}{3}$ cm. Find the length of the diameter in the circle.

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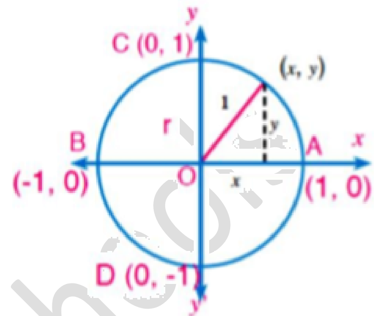
Lesson 3

Trigonometric function

The unit Circle:



In any orthogonal coordinates system:
A circle of center at the origin point and of radius equals one unit is called a unit circle.

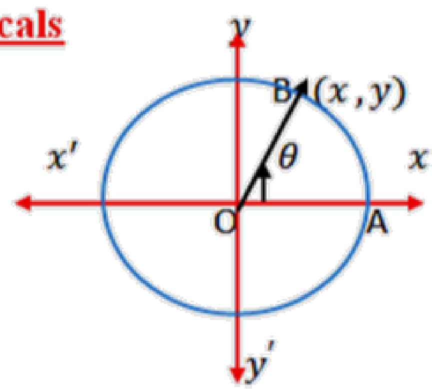
Remarks:

1) The unit circle intersect the x – axis at the two points (1, 0), (-1, 0) and intersect the y – axis at the two points (0, 1), (0, -1).

2) If the point (x, y) ∈ the unit circle, then

*$x^2 + y^2 = 1$ from Pythagoras' theorem. Where $x \in [-1, 1], y \in [-1, 1]$

The basic Trigonometric functions and their reciprocals



If We draw the directed angle AOB in the standard position and its terminal side intersects the unit circle At the point B (x, y) and if $m(\angle AOB) = \theta$, then

1) The basic trigonometric functions of the angle whose measure θ are :

(1) cosine of the angle $= x$ – coordinate of the point B so $\cos\theta = x$

(2) Sine of the angle $= y$ – coordinate of the point B so $\sin\theta = y$

(3) Tangent of the angle $= \dfrac{y\text{–coordinate of the point B}}{x\text{–coordinate of the point B}}$

So $\tan\theta = y/x = \sin\theta/\cos\theta$, where $x \neq 0$

Notice that: The coordinates of the point B (x, y) can be written as $(\cos\theta, \sin\theta)$

2) The reciprocals of the basic Trigonometric function for the angle of measure θ are:

(1) The Secant of the angle (sec) = $\dfrac{1}{x\text{−coordinate of the point B}}$

$$\text{So } \sec\theta = \frac{1}{x} = \frac{1}{\cos\theta} \text{ , where } x \neq 0$$

(2) The Cosecant of the angle (csc) = $\dfrac{1}{y\text{−coordinate of the point B}}$

$$\text{So } \csc\theta = \frac{1}{y} = \frac{1}{\sin\theta} \text{ , where } y \neq 0$$

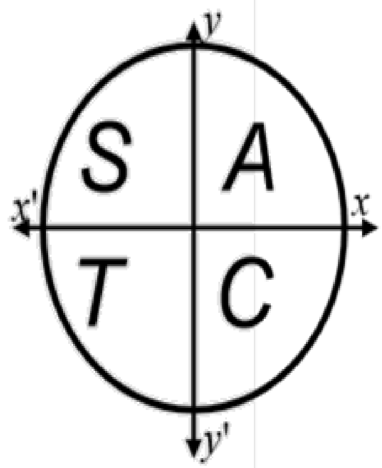
(3) The Cotangent of the angle = $\dfrac{x\text{−coordinate of the point B}}{y\text{−coordinate of the point B}}$

$$\text{So } \cot\theta = \frac{x}{y} = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta} \text{ , where } y \neq 0$$

Signs of trigonometric functions: (ASTC)

We can summarize signs of the trigonometric functions in the following table:

Quadrant	Sign of Cos, sec	Sign of Sin, csc	Sign of tan, cot
First $]0, \frac{\pi}{2}[$	+	+	+
Second $]\frac{\pi}{2}, \pi[$	–	+	–
Third $]\pi, \frac{3\pi}{2}[$	–	–	+
Forth $]\frac{3\pi}{2}, 2\pi[$	+	–	–



Exercises (3)

Choose the correct answer from those given :

(1) If θ is the measure of an angle in the standard position , its terminal side intersects the unit circle at the point $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, then sin θ = ...

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

(2) If sin θ = $\frac{1}{\sqrt{2}}$, where θ is the measure of a positive acute angle , then the measure of angle θ = ...

(a) 30° (b) 60° (c) 45° (d) 90°

(3) 📖 If sin θ = –1 , cos θ = 0 , then the measure of angle θ = ...

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2 π

(4) 📖 If csc θ = 2 , where θ is a positive acute angle , then the measure of angle θ = ...

(a) 15° (b) 30° (c) 45° (d) 60°

(5) If tan θ = 1 , where θ is a positive acute angle , then the measure of angle θ = ...

(a) 60° (b) 30° (c) 45° (d) 90°

(6) 📖 If cos θ = $\frac{\sqrt{3}}{2}$, where θ is a positive acute angle , then sin θ = ...

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

(7) $\cot^2 30° - \sec^2 60° + \csc^2 45°$ = ...

(a) 1 (b) 0 (c) –1 (d) 2

1] Determine the signs of the following trigonometric functions:

(1) $\tan 410^0$ (2) $\sec 265^0$ (3) $\cot 32\pi/3$ (4) $\cot -3\pi/4$

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2]If the terminal side of the directed angle whose measure is θ in the standard position intersects the unit circle at the point A $(-\frac{3}{4}, \frac{-\sqrt{7}}{4})$:

(1) Determine the quadrant in which the angle θ lies.

(2) Find all trigonometric functions of the angle θ

……………………………………………………………………………………………
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3] If θ is the measure of the directed angle in the standard position and B is the intersection point of its terminal side with the unit circle , then find all trigonometric

functions of the angle θ in each of the following cases:

(1) B $(-x , x)$, $x > 0$ (2) B $(\frac{3a}{2} \ , -2a)$ where $3\pi/2 < \theta < 2\pi$

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4] Find the value of each of :

(1) $\tan^2 30^0 + 2\sin^2 45^0 + \cos^2 90^0$ (2) $\cos\frac{\pi}{2}\cos 0^0 + \sin\frac{3\pi}{2}\sin\frac{\pi}{2}$

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……………………………………………………………………………………………

5] Prove each of the following equalities :

(1) $\cos\frac{\pi}{2} = \cos^2\frac{\pi}{4} - \sin^2\frac{\pi}{4}$ (2) $\sin 60^0 \cos 30^0 - \cos 60^0 \sin 30^0 = \sin\frac{\pi}{4}$

……………………………………………………………………………………………
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Lesson 4

Related angles

Definition of the related angles:

They are two angles the difference between their measures or the sum of their measures equal a whole number of right angle.

The relation between trigonometric functions of related angles

2^{nd} quadrant , sin , csc are (+ve)

$(180^0 - \theta)$, $(90^0 + \theta)$

1^{st} quadrant, All functions are (+ve)

θ , $(90^0 - \theta)$

3^{rd} quadrant , tan , cot are (+ve)

$(270^0 - \theta)$, $(180^0 + \theta)$

4^{th} quadrant cos , sec are (+ve)

$(270^0 + \theta)$, $(360^0 - \theta)$, $-\theta$

y, y', x, x', O

1 Relation between trigonometric functions of related angles of measures θ^0 $(90^0 - \theta^0)$

$\sin(90^0 - \theta) = \cos\theta$, $\csc(90^0 - \theta) = \sec\theta$

$\cos(90^0 - \theta) = \sin\theta$, $\sec(90^0 - \theta) = \csc\theta$

$\tan(90^0 - \theta) = \cot\theta$, $\cot(90^0 - \theta) = \tan\theta$

2 Relation between trigonometric functions of related angles of measures θ^0, $(90^0 + \theta^0)$

$\sin(90^0 + \theta) = \cos\theta$, $\csc(90^0 + \theta) = \sec\theta$

$\cos(90^0 + \theta) = -\sin\theta$, $\sec(90^0 + \theta) = -\csc\theta$

$\tan(90^0 + \theta) = -\cot\theta$, $\cot(90^0 + \theta) = -\tan\theta$

3 Relation between trigonometric functions of related angles of measures θ^0, $(180^0 - \theta^0$

$\sin(180^0 - \theta) = \sin\theta$, $\csc(180^0 - \theta) = \csc\theta$

$\cos(180^0 - \theta) = -\cos\theta$, $\sec(180^0 - \theta) = -\sec\theta$

$\tan(180^0 - \theta) = -\tan\theta$, $\cot(180^0 - \theta) = -\cot\theta$

4 Relation between trigonometric functions of related angles of measures θ^0, $(180^0+\theta^0)$

$$\sin(180^0+\theta) = -\sin\theta \qquad , \csc(180^0+\theta) = -\csc\theta$$
$$\cos(180^0+\theta) = -\cos\theta \qquad , \sec(180^0+\theta) = -\sec\theta$$
$$\tan(180^0+\theta) = \tan\theta \qquad , \cot(180^0+\theta) = \cot\theta$$

5 Relation between trigonometric functions of related angles of measures θ^0, $(270^0-\theta^0)$

$$\sin(270^0-\theta) = -\cos\theta \qquad , \csc(270^0-\theta) = -\sec\theta$$
$$\cos(270^0-\theta) = -\sin\theta \qquad , \sec(270^0-\theta) = -\csc\theta$$
$$\tan(270^0-\theta) = \cot\theta \qquad , \cot(270^0-\theta) = \tan\theta$$

6 Relation between trigonometric functions of related angles of measure θ^0, $(270^0+\theta^0)$

$$\sin(270^0+\theta) = -\cos\theta \qquad , \csc(270^0+\theta) = \sec\theta$$
$$\cos(270^0+\theta) = \sin\theta \qquad , \sec(270^0+\theta) = \csc\theta$$
$$\tan(270^0+\theta) = -\cot\theta \qquad , \cot(270^0+\theta) = -\tan\theta$$

7 Relation between trigonometric functions of related angles of measures θ^0, $(360^0-\theta^0)$

$$\sin(360^0-\theta) = -\sin\theta \qquad , \csc(360^0-\theta) = -\csc\theta$$
$$\cos(360^0-\theta) = \cos\theta \qquad , \sec(360^0-\theta) = \sec\theta$$
$$\tan(360^0-\theta) = -\tan\theta \qquad , \cot(360^0-\theta) = -\cot\theta$$

Remark:

$$\sin(-\theta) = -\sin\theta \qquad , \cos(-\theta) = \cos\theta \qquad , \tan(-\theta) = -\tan\theta$$

Exercises (4)

Choose the correct answer :

(1) If $\sin\theta = \cos 2\theta$, $\theta \in \left]0 , \frac{\pi}{2}\right[$, then $\sin 3\theta =$ ....

(a) $\frac{1}{2}$ (b) 1 (c) zero (d) $\frac{\sqrt{3}}{2}$

(2) If $\tan\theta = \cot 2\theta$, $0° < \theta < 90°$, then $\sin\theta + \cos 2\theta =$ ....

(a) 1 (b) -1 (c) 2 (d) $\frac{1}{4}$

(3) If $\sin\alpha = \cos\beta$ where α and β are two acute angles , then $\tan(\alpha + \beta) =$ ....

(a) $\frac{1}{\sqrt{3}}$ (b) 1 (c) $\sqrt{3}$ (d) undefined

(4) If $\sin 2\theta = \cos 4\theta$ where θ is a positive acute angle , then $\tan(90° - 3\theta) =$ ....

(a) -1 (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\sqrt{3}$

(5) If $5\cos(90° - \theta) = 4$, $0° < \theta < 90°$, then $\sin\theta =$ ....

(a) $\frac{5}{4}$ (b) $\frac{-3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5}$

(6) If $\cot(90° + \theta) + 1 = 0$ where $0° < \theta < 90°$, then $\cos 4\theta =$ ....

(a) $\frac{1}{2}$ (b) 1 (c) zero (d) -1

(7) If $\cos(90° + \theta) + \sin(90° - 2\theta) = 0$, where $\theta \in \left]0 , \frac{\pi}{4}\right[$, then $\sin 2\theta =$ ....

(a) $\frac{1}{2}$ (b) 1 (c) zero (d) $\frac{\sqrt{3}}{2}$

(8) If $\cos(270° - \theta) = \frac{1}{2}$ where θ is the measure of the smallest positive angle , then $\theta =$ ....

(a) 30° (b) 150° (c) 210° (d) 330°

(9) If $\tan\theta = \frac{-5}{12}$, $\cos\theta < 0$, then $\csc\theta =$ ....

(a) $\frac{5}{13}$ (b) $\frac{-5}{13}$ (c) $\frac{13}{5}$ (d) $\frac{-13}{5}$

(10) If $\sin\theta = -\frac{1}{2}$, $\tan\theta > 0$, then $\theta =$ ....

(a) 30° (b) 150° (c) 210° (d) 330°

Complete the following :

(1) $\tan 42° = \cot$ ……….°

(2) $\csc 13° = \sec$ ……….°

(3) $\sin 25° = \cos$ ……….°

(4) $\cos 67° = \sin$ ………. °

(5) $\cos(90° - \theta) =$ ……….

(6) $\cot(90° + \theta) =$ ……….

(7) $\csc(360° - \theta) =$ ……….

(8) $\tan(180° - \theta) =$ ……….

(9) $\sec(270° - \theta) =$ ……….

(10) $\sin(-\theta) =$ ……….

(11) $\cos(\theta - 90°) =$ ……….

(12) $\csc(\theta - 270°) =$ ……….

(13) $\dfrac{\sec 105°}{\csc 15°} =$ ……….

(14) $\dfrac{\sin 15°}{\tan 20°} \times \dfrac{\cot 70°}{\cos 75°} =$ ……….

(15) $\tan 120° = \tan(90° +$ ……….$) = -\cot$ ……….° = ……….

(16) $\sin 300° = \sin(360° -$ ……….$) = -\sin$ ……….° = ……….

(17) $\cos\theta + \cos(180° - \theta) =$ ……….

(18) $\sin\theta + \cos(270° + \theta) =$ ……….

(19) If α , β are the measures of two complementary angles and $\sin\alpha = \frac{3}{5}$, then $\cos\beta =$ ……….

(20) If $\sin 2\theta = \cos 3\theta$, $0° < \theta < 90°$, then $\theta =$ ……….°

(21) If $\tan 2\theta = \cot 3\theta$ where $\theta \in \left]0 , \frac{\pi}{2}\right[$, then $\theta =$ ………. rad

(22) If $\cos\theta = \sin 2\theta$ where θ is positive acute angle , then $\sin 3\theta =$ ……….

(23) If $\sin\theta = \sin(90° - \theta)$, then $\tan\theta =$ ……….

(24) If $\csc\theta = \frac{2}{\sqrt{3}}$, $\theta \in \left]0 , \frac{3\pi}{2}\right[$, then $\theta =$ ………. ° or $\theta =$ ………. °

(25) If $\cot 2\theta - \tan\theta = 0$ where θ is the measure of a positive acute angle , then $\theta =$ ……….°

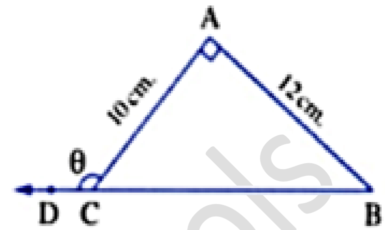
By using the calculator , choose the correct answer :

(1) In the opposite figure :

$D \in \overrightarrow{BC}$, AC = 10 cm. , AB = 12 cm. , then cot θ = ...

(a) $\frac{6}{5}$ (b) $-\frac{6}{5}$

(c) $\frac{5}{6}$ (d) $-\frac{5}{6}$

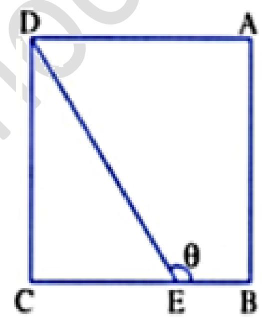


(2) In the opposite figure :

ABCD is a square , CE = 2 BE , then tan θ = ...

(a) $-\frac{3}{2}$ (b) $-\frac{2}{3}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

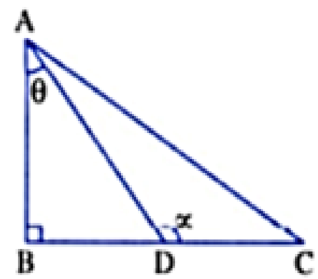


(3) In the opposite figure :

Δ ABC is a right-angled triangle at B , tan θ = $\frac{3}{4}$,
then cos α = ...

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$

(c) $\frac{4}{5}$ (d) $-\frac{3}{5}$

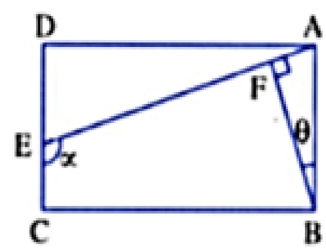


(4) In the opposite figure :

ABCD is a rectangle , tan θ = $\frac{1}{3}$, $\overline{BF} \perp \overline{AE}$,
then cot α = ...

(a) $\frac{1}{3}$ (b) $\frac{3}{4}$

(c) $-\frac{1}{3}$ (d) $\frac{2}{3}$

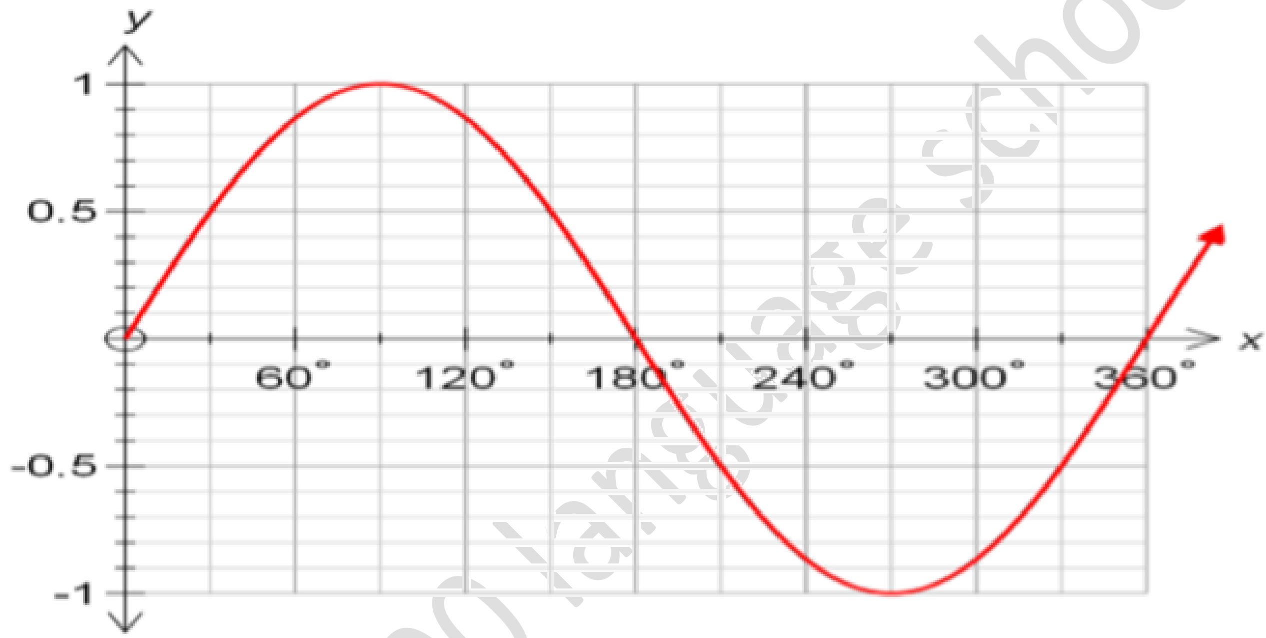


Lesson 5

Graphing trigonometric function

First sine function: $f : f(\theta) = \sin\theta$

θ	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{8\pi}{6}$	$\frac{9\pi}{6}$	$\frac{10\pi}{6}$	$\frac{11\pi}{6}$	π
$\sin\theta$	0	0.5	0.87	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0



From the previous, we can deduce that :

Properties of the sine function in the form: $f : f(\theta) = \sin\theta$

1)The domain of the sine functionis $]-\infty, \infty[$

2) *The maximum value of the function is 1 and it happens when $\theta = \frac{\pi}{2} + 2n\pi$

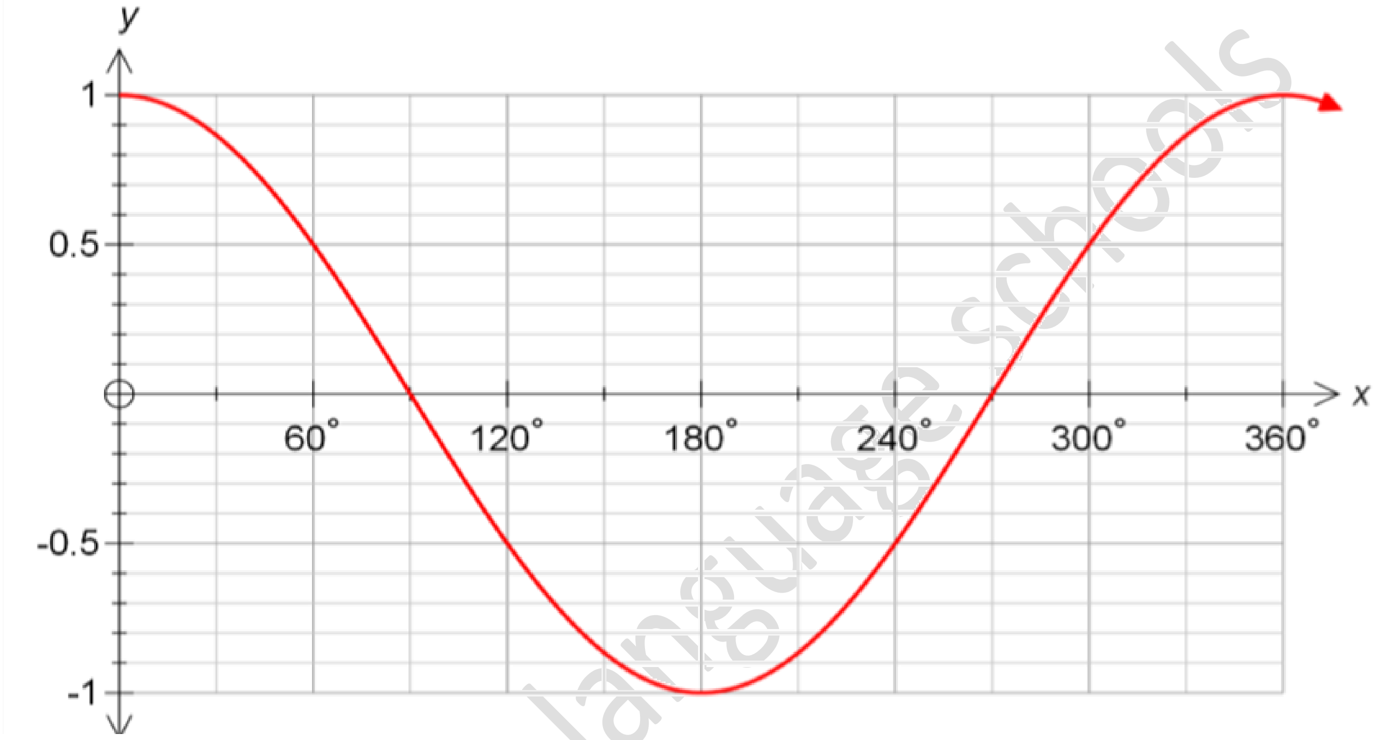
*The minimum value of the function is -1 and it happens when $\theta = \frac{3\pi}{2} + 2n\pi$ where $n \in Z$

*The range of the function $= [-1, 1]$

3) The function is periodic and its period is 2π (360^0)

Second cosine function : $f: f(\theta) = \cos\theta$

θ	0	$\frac{\pi}{2}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{8\pi}{6}$	$\frac{9\pi}{6}$	$\frac{10\pi}{6}$	$\frac{11\pi}{6}$	2π
$\cos\theta$	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0	0.5	0.87	1



From the previous, we can deduce that:

Properties of the cosine function in the form: $f : f(\theta)$

1) The domain of the cosin function is $]-\infty, \infty[$

2) *The maximum value of the function is 1and it happens when $\theta = \frac{\pi}{2} + 2n\pi$

*The Minimum value of the function is – 1 and it happens when

$\theta = \frac{3\pi}{2} + 2n\pi$ where $n \in Z$ *The range of the function = [-1, 1]

3) The function is periodic and its its period is 2π (360^0)

Note: Each of the two functions: $y = a \sin b\theta$, $y = a \cos b\theta$is periodic on its period is $2\pi/|b|$ and its range [- a, a] where a is positive.

Exercises (5)

1) Complete the following :

(1) The range of the function f where $f(\theta) = \sin\theta$ is ……………

(2) The range of the function f where $f(\theta) = 2\sin\theta$ is …………

(3) If : $f(x) = 4\sin\theta$, then the range of the function is…………

(4) If : $f(x) = \cos 5\theta$, then the range of the function is…………

(5) The maximum value of the function $f : f(\theta) = 4\sin\theta$ is…………

(6) The minimum value of the function f where : $f(\theta) = 5\sin\theta$ is………

(7) function $f : f(\theta) = 2\sin 4\theta$ is a periodic function and its period =…..

2] Find the maximum and minimum values , Then calculate the range of each of the following functions :

(1) $y = \sin\theta$ (2) $y = \frac{1}{2}\sin\theta$ (3) $y = 3\cos\theta$ (4) $y = \frac{3}{2}\cos\theta$

……………………………………………………………………………………
……………………………………………………………………………………
……………………………………………………………………………………

3 Multiple choice:

1. If $\sin\theta = 0.4325$ where θ is a positive acute angle, then $m(\angle\theta)$ equals ……
   - A 25.626°
   - B 64.347°
   - C 32.388°
   - D 46.316°

2. If $\tan\theta = 1.8$ and $90° \leqslant \theta \leqslant 360°$, then $m(\angle\theta)$ equals ……
   - A 60.945°
   - B 119.055°
   - C 240.945°
   - D 299.055°

4 Use the degree measure to find the smallest positive angle which satisfies each of the following:

A $\sin^{-1} 0.6$ ……………

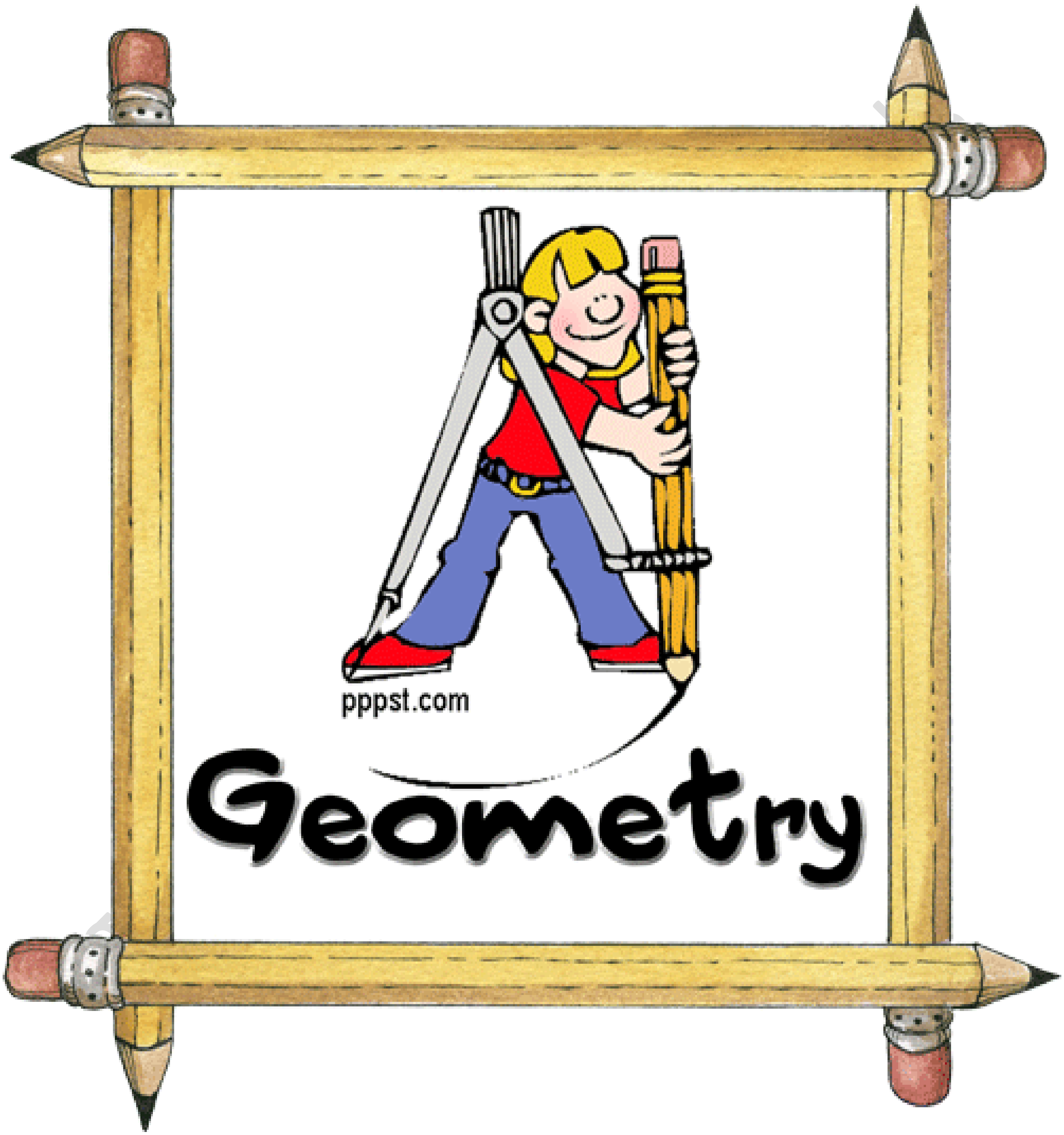
B $\cos^{-1} 0.436$ ……………

C $\tan^{-1} 1.4552$ ……………

D $\sec^{-1}(-2.2364)$ ……………

E $\cot^{-1} 3.6218$ ……………

F $\csc^{-1}(-1.6004)$ ……………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Geometry

Unit 3

Lesson (1)

Similarity of polygons

Definition

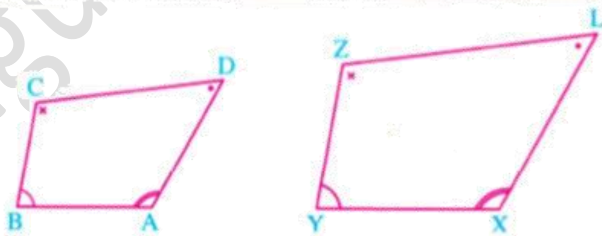
Two polygons M_1 and M_2 (having the same number of sides) are said to be similar if the following two conditions satisfied together :

1. Their corresponding angles are congruent.
2. The lengths of their corresponding sides are proportional.

In this case , we shall write : the polygon $M_1 \sim$ the polygon M_2 , that means the polygon M_1 is similar to the polygon M_2

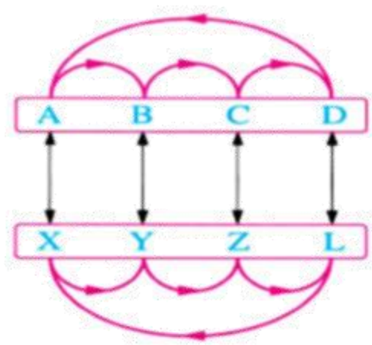
In the opposite figure , if :

1. $m(\angle A) = m(\angle X)$, $m(\angle B) = m(\angle Y)$, $m(\angle C) = m(\angle Z)$, $m(\angle D) = m(\angle L)$



2. $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$, then the polygon ABCD $\sim$ the polygon XYZL

If the polygon ABCD $\sim$ the polygon XYZL , then :

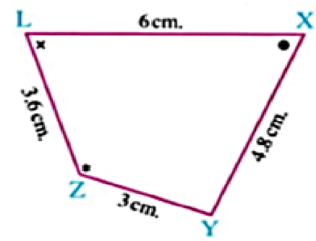


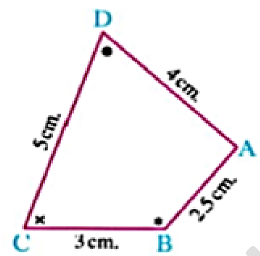
1. $m(\angle A) = m(\angle X)$, $m(\angle B) = m(\angle Y)$, $m(\angle C) = m(\angle Z)$, $m(\angle D) = m(\angle L)$
2. $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$

Example

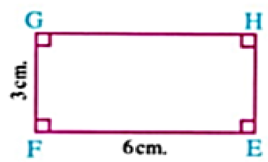
Show which of the following pairs of polygons are similar , showing the reason and if they are similar , determine the similarity ratio :

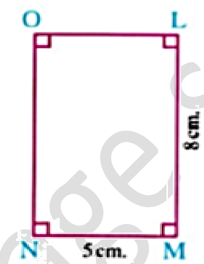
1





2





Similarity ratio of two polygons

Let K be the similarity ratio of polygon M_1 to polygon M_2

If: $K > 1$ **then** polygon M_1 is an enlargement of polygon M_2

$0 < K < 1$ **then** polygon M_1 is a shrinking of polygon M_2

$K = 1$ **then** polygon M_1 is congruent to polygon M_2

In general: you can use the similarity ratio in calculation of the dimensions of similar figures.

Golden ratio

rectangle ABCD ~ rectangle EFBC

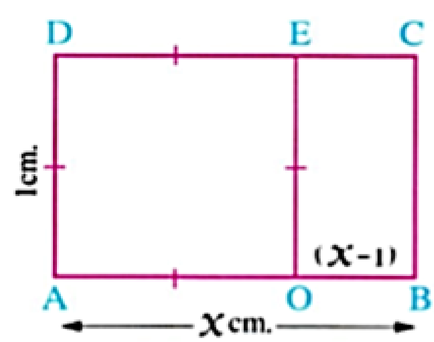
$$\frac{AB}{EF} = \frac{BC}{FB} \longrightarrow \frac{x}{1} = \frac{1}{x-1}$$

$$\therefore x^2 - x - 1 = 0$$

by solving the quadratic equation, we get:

$$x = \frac{1+\sqrt{5}}{2} \quad , \quad x = \frac{1-\sqrt{5}}{2} < 0 \qquad \text{refused}$$

$$\simeq 1.618$$



The golden ratio is 1.618 : 1 approximately.

Note

All golden rectangles are similar.

Exercises (1)

(1) Two polygons of the same number of sides are similar if ....

(2) If the scale factor of similarity of two polygons = 1 , then the two polygons are ....

(3) Two similar polygons , the ratio between the lengths of two corresponding sides in them is 2 : 3 , if the perimeter of the smaller is 14 cm. , then the perimeter of the bigger is .... cm.

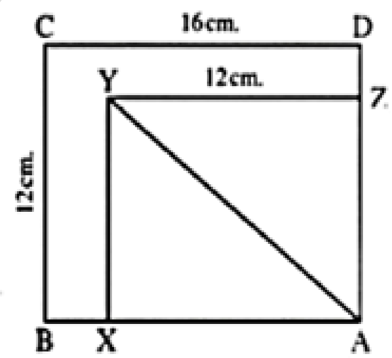
(4) In the opposite figure :

If rectangle ABCD ~ rectangle AXYZ ,

DC = 16 cm. ,

BC = ZY = 12 cm.

, then AY = ... cm.



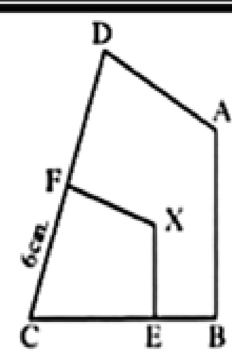
In the opposite figure :

Polygon ABCD ~ polygon XECF

(1) Prove that : $\overline{AB} \parallel \overline{XE}$

(2) If XE = $\frac{1}{2}$ AB , CF = 6 cm.

, find the length of : $\overline{FD}$



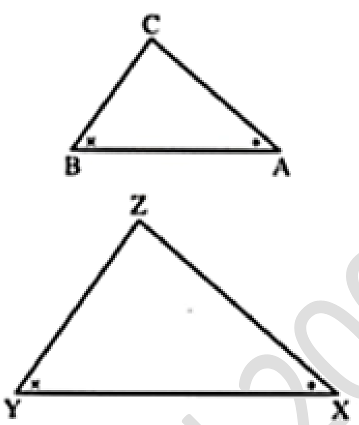
Lesson (2)

Similarity of triangles

The two triangles are similar

First case

If two angles of one triangle are congruent to their corresponding angles of another triangle.



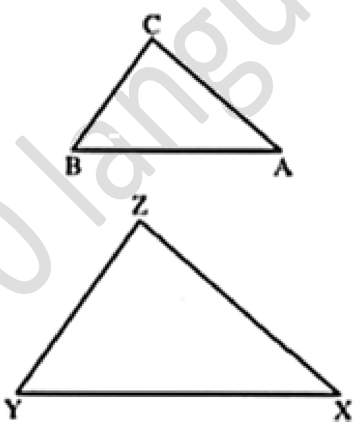
If $\angle A \equiv \angle X$

$, \angle B \equiv \angle Y$

then $\Delta ABC \sim \Delta XYZ$

Second case

If the side lengths of two triangles are in proportion.

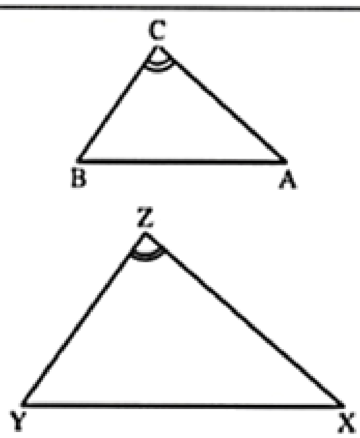


If $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$

, then $\Delta ABC \sim \Delta XYZ$

Third case

If an angle of one triangle is congruent to an angle of another triangle and lengths of the sides including those angles are in proportion.



If $\angle C \equiv \angle Z$,

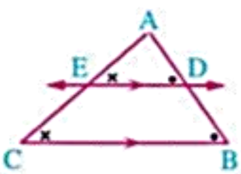
$\frac{CA}{ZX} = \frac{CB}{ZY}$

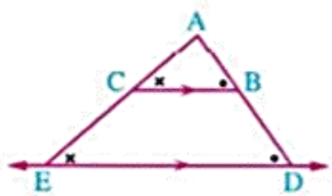
, then $\Delta ABC \sim \Delta XYZ$

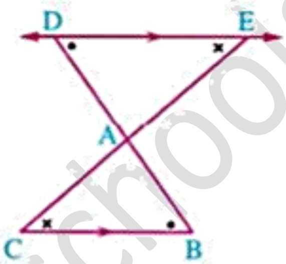
Corollary (1)

If a line is drawn parallel to one side of a triangle and intersects the other two sides or the lines containing them , then the resulting triangle is similar to the original triangle.

In each of the following figures :







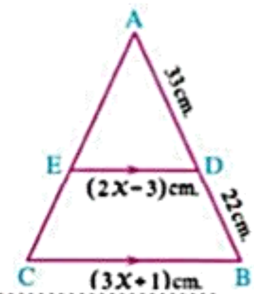
If $\overleftrightarrow{DE} \parallel \overline{BC}$ and intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ at D and E respectively , then $\Delta ABC \sim \Delta ADE$

Example In the opposite figure :

$\overline{DE} \parallel \overline{BC}$, AD = 33 cm. , DB = 22 cm.

, DE = $(2x - 3)$ cm. and BC = $(3x + 1)$ cm.

1 Prove that : $\Delta ADE \sim \Delta ABC$ **2** Find the value of : x



Solution..

....

....

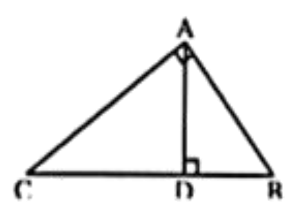
Corollary (2)

In any right-angled triangle , the altitude to the hypotenuse separates the triangle into two triangles which are similar to each other and to the original triangle.

In the opposite figure :

If ΔABC is a right-angled triangle at A and $\overline{AD} \perp \overline{BC}$

, then $\Delta DBA \sim \Delta DAC \sim \Delta ABC$ and from this we can deduce that :



- $(AB)^2 = BD \times BC$
- $(AC)^2 = CD \times CB$
- $(AD)^2 = BD \times DC$
- $AD \times BC = AB \times AC$

Example

ABCD is a rectangle , Draw $\overrightarrow{DF} \perp \overline{AC}$ to cut $\overline{AC}$ in E , $\overline{BC}$ in F

Prove that : The area of the rectangle ABCD $= \sqrt{AE \times AC \times DE \times DF}$

Solution……………………………………………………………………………………

……………………………………………………………………………………………

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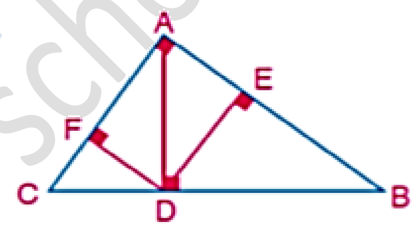
Example

In the figure opposite: ABC is a right angled triangle at A,

$\overline{AD} \perp \overline{BC}$, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{AC}$. Prove that:

(A) $\Delta ADE \sim \Delta CDF$

(B) Area of rectangle AEDF $= \sqrt{AE \times EB \times AF \times FC}$



Solution……………………………………………………………………………………

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Example Complete :

1) Two polygons are similar if …………… and ……………

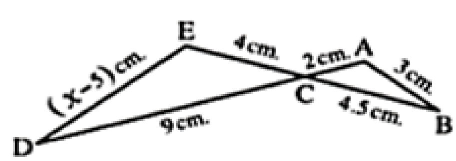
2) If the side lengths of two triangles are in proportion , then the two triangles are ……………

3) In any right-angled triangle , the altitude to the hypotenuse separates the triangle into two triangles which are …………… to each other and to the original triangle.

4) Two polygons are similar if …………… , ……………

5) **In the opposite figure :**

$x =$ ……………



(a) 5 cm. (b) 11 cm.

(c) 12 cm. (d) 14 cm.

6) Which of the following polygons are always similar ?

(a) Two rectangles. (b) Two isosceles triangles.

(c) Two rhombuses. (d) Two equilateral triangles.

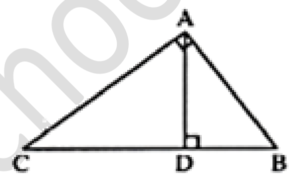
Exercises (2)

1 **Choose the correct answer from those given :**

(1) Two similar rectangles , the two dimensions of the first are 12 cm. , 8 cm. and the perimeter of the second is 60 cm. , then the length of the second rectangle = ……………

(a) 12 cm. (b) 18 cm. (c) 24 cm. (d) 16 cm.

(2) In the opposite figure :

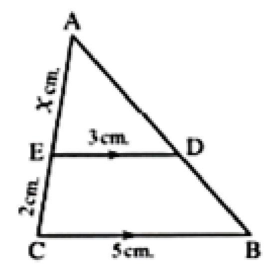


Which of the following expressions is wrong ?

(a) $(AB)^2 = BD \times DC$ (b) $(AC)^2 = CD \times CB$

(c) $(AD)^2 = DB \times DC$ (d) $AB \times AC = BC \times AD$

(3) In the opposite figure :



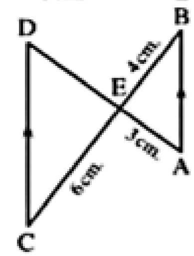
If $\overline{DE} \parallel \overline{BC}$

, then $X =$ ……………

(a) 6 cm. (b) 3 cm.

(c) 5 cm. (d) 1.2 cm.

(4) In the opposite figure :



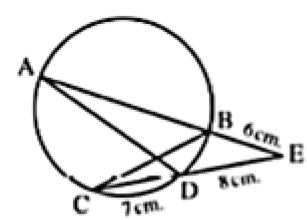
$\overline{AB} \parallel \overline{CD}$, AE = 3 cm.

, BE = 4 cm. , EC = 6 cm.

, then ED = ……………

(a) 4 cm. (b) 6 cm. (c) 3 cm. (d) $4\frac{1}{2}$ cm.

2 **In the opposite figure :**



$\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$ where E is outside the circle.

If EB = 6 cm. , ED = 8 cm. , DC = 7 cm.

(1) Prove that : $\Delta ADE \sim \Delta CBE$

(2) Find the length of : $\overline{AE}$

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

Lesson (3)

Relation between the areas of two similar polygons

Theorem (3)

The ratio between the areas of the surfaces of two similar triangles equals the square of the ratio between the lengths of any two corresponding sides of the two triangles.

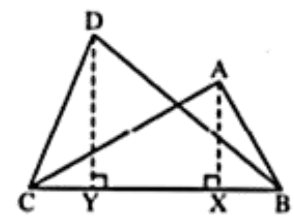
Theorem (4)

The ratio between the areas of the surfaces of two similar polygons equals the square of the ratio between the lengths of any two corresponding sides of the polygons.

Remarks

- The ratio of the areas of two triangles having a common base equals the ratio of the two heights of the two triangles.

In the opposite figure :



$\overline{BC}$ is a common base of $\Delta\Delta$ ABC , DBC

$$\therefore \frac{a(\Delta ABC)}{a(\Delta DBC)} = \frac{\frac{1}{2}BC \times AX}{\frac{1}{2}BC \times DY} = \frac{AX}{DY}$$

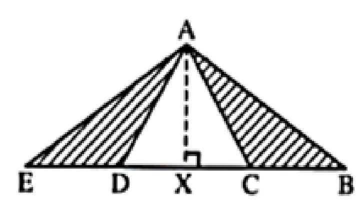
Notice that : It is not necessary that the two triangles are similar.

Notice that : It is not necessary that the two triangles are similar.

Remarks

The ratio of the areas of two triangles having a common height equals the ratio of the lengths of two bases of the two triangles.

In the opposite figure :



AX is a common height for $\Delta\Delta$ ABC , ADE

$$\therefore \frac{a(\Delta ABC)}{a(\Delta ADE)} = \frac{\frac{1}{2}BC \times AX}{\frac{1}{2}DE \times AX} = \frac{BC}{DE}$$

Notice that : It is not necessary that the two triangles are similar.

Exercises (3)

1 **Complete the following :**

(1) If two angles in one triangle are congruent to their corresponding angles in another triangle , then the two triangles are ....

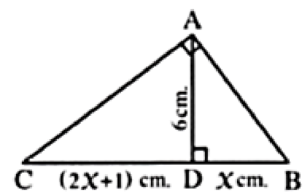
(2) If the ratio between the perimeters of two similar polygons is 4 : 9 , then the ratio between their areas is ....

(3) In the opposite figure :

$x =$ ....

(4) In the opposite figure :

$x =$ ....



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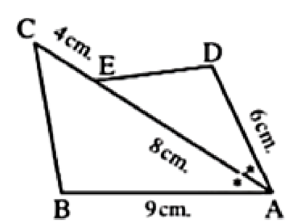
2 **In the opposite figure :**

$\overrightarrow{AE}$ bisects $\angle$ DAB

, area of Δ ADE = 12 cm^2 .

Find the area of : Δ ABC



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3 ABCD , XYZL are two similar polygons. If M is the midpoint of $\overline{BC}$

N is the midpoint of $\overline{YZ}$, AM = 4 cm. , XN = 9 cm.

, prove that :

area of polygon ABCD : area of polygon XYZL = 16 : 81

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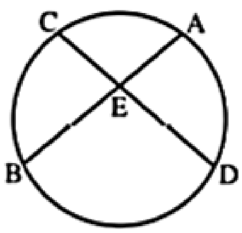
Lesson (4)

Application of similarity in the circle

Well known problem

If $\overline{AB}$, $\overline{CD}$ are two chords in a circle

, $\overline{AB} \cap \overline{CD} = \{E\}$

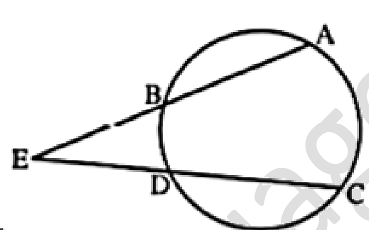


then

$EA \times EB = EC \times ED$

If $\overline{AB}$ and $\overline{CD}$ are two chords in a circle

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$

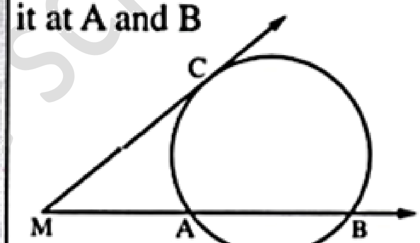


then

$EA \times EB = EC \times ED$

Corollary

If M is a point outside the circle , $\overrightarrow{MC}$ touches the circle at C , $\overrightarrow{MB}$ intersects it at A and B



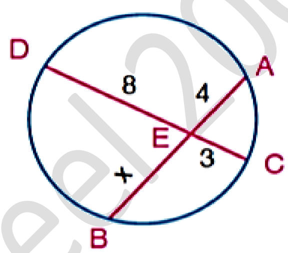
then

$(MC)^2 = MA \times MB$

Example

Use the calculator or mental math to find the numerical value of x in each of the following figures. (lengths are measured in centimetres)

A



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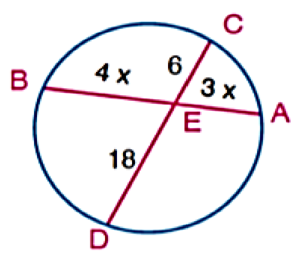
B



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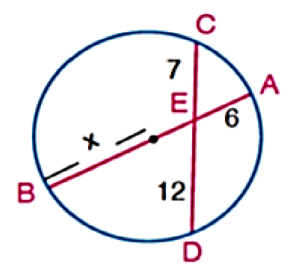
C



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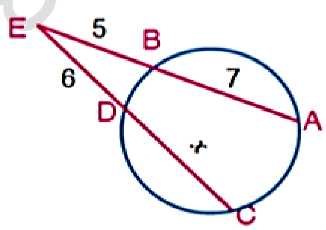
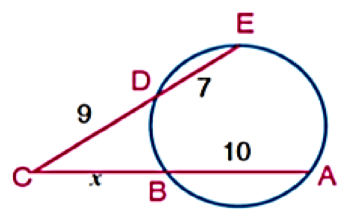
D



………………………………………

E



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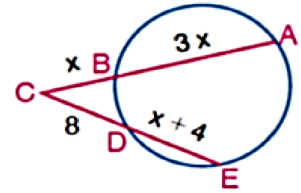
F



………………………………………

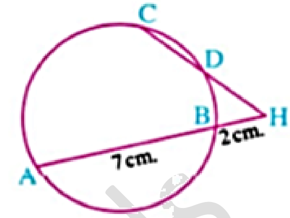
Example

In the opposite figure :

$\overrightarrow{AB} \cap \overrightarrow{CD} = \{H\}$, HB = 2 cm.

, AB = 7 cm. , if $\frac{HD}{HC} = \frac{1}{2}$

, **find the length of :** $\overline{HC}$



Solution……………………………………………………………………………………………

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Example

In the opposite figure :

$\overline{BC}$ is a tangent to a circle M

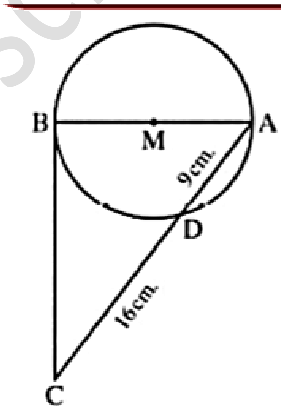
, $\overline{AB}$ is a diameter

, $\overline{CA}$ intersects the circle at D

Find :

(1) The length of $\overline{CB}$

(2) The area of the circle



Solution……………………………………………………………………………………………

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Example

In the opposite figure :

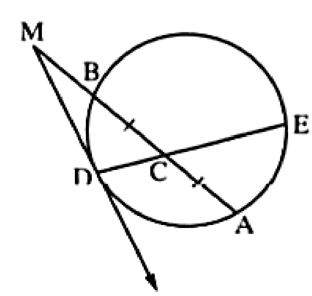
$\overline{AB} \cap \overline{DE} = \{C\}$

, CA = CB , CD = 2 cm. , CE = 8 cm.

, $\overline{MD}$ is a tangent to the circle

, $MB = \frac{1}{2}$ AB

Find the length of $\overline{MD}$



Solution……………………………………………………………………………………………

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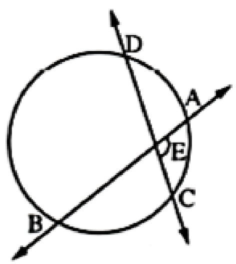
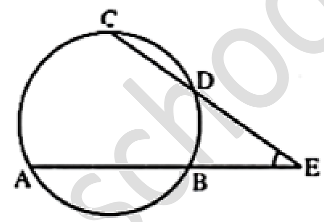
Secant , tangent and measures of angles

1 The measure of an angle formed by two chords that intersect inside a circle is equal to half the sum of the measures of the intercepted arcs.



$m(\angle AEC) = \frac{1}{2}[m(\overset{\frown}{AC}) + m(\overset{\frown}{BD})]$

2 The measure of an angle formed by two secants drawn from a point outside a circle is equal to half the positive difference of the measures of the intercepted arcs.



$m(\angle E) = \frac{1}{2}[m(\overset{\frown}{AC}) - m(\overset{\frown}{BD})]$

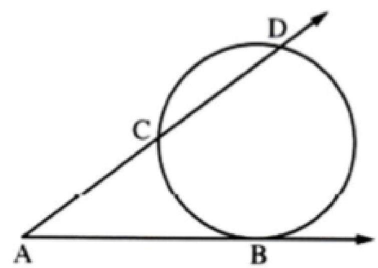
Example

In the opposite figure :

If $m(\angle A) = 50°$

and $m(\overset{\frown}{BC}) = 60°$

find $m(\overset{\frown}{BD})$



Solution..

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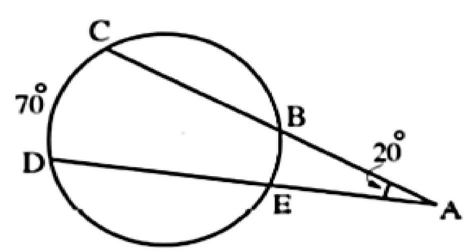
Example

In the opposite figure :

$m(\angle A) = 20°$, $m(\overset{\frown}{DC}) = 70°$

, then $m(\overset{\frown}{BE}) =$ ....



Solution..

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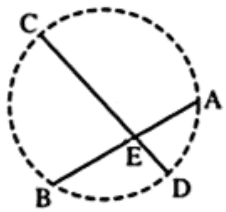
Converse of the well known and the corollary

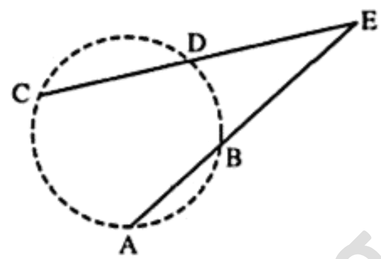
Converse of the well known problem

If $\overline{AB} \cap \overline{CD} = \{E\}$,
A , B , C , D and E are distinct points and
$EA \times EB = EC \times ED$



, then the points A , B , C and D lie on the same circle.

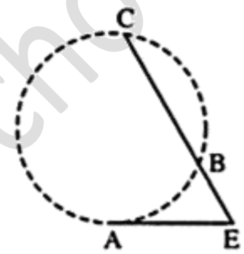
If $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$,
A , B , C , D and E are distinct points and
$EA \times EB = EC \times ED$



, then the points A , B , C and D lie on the same circle.

Converse of the corollary

If $E \in \overrightarrow{CB}$, $E \notin \overline{BC}$,
and $(EA)^2 = EB \times EC$



, then $\overline{EA}$ is a tangent segment to the circle which passes throught the points A , B and C

Example

In which of the following figures , do the points A , B , C and D lie on the same circle ?

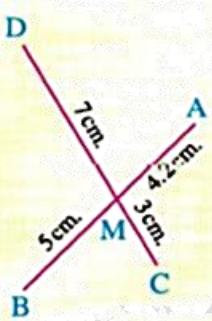


Fig. (1)

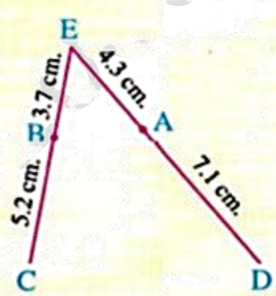


Fig. (2)

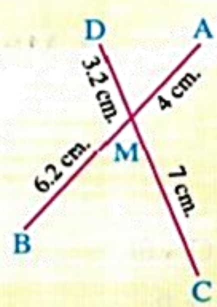


Fig. (3)

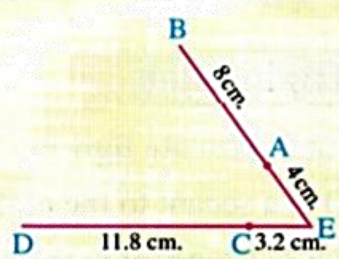


Fig. (4)

Solution……………………………………………………………………………………

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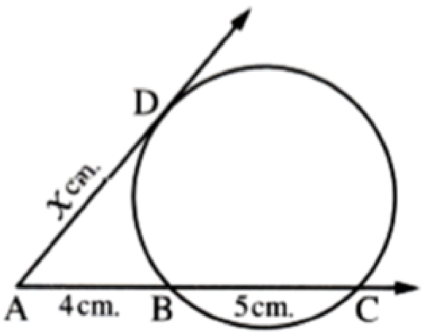
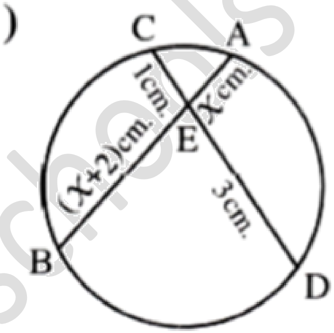
Exercises (4)

1 **Find the numerical value of X in each of the following figures :**

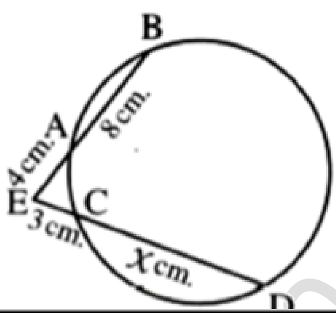
(1)



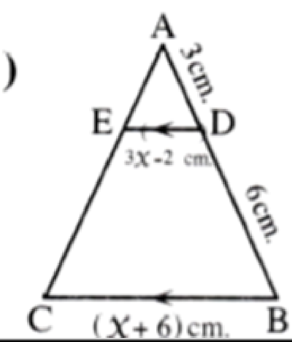
(2)



(3)

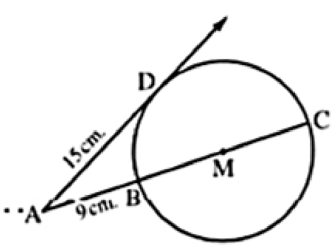


(4)



2 In the opposite figure : $\overrightarrow{AD}$ is a tangent to the circle M at D where AD = 15 cm. , if AB = 9 cm.

Calculate the radius length of the circle.



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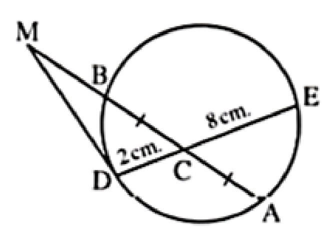
3 In the opposite figure : $\overline{AB} \cap \overline{DE} = \{C\}$, CA = CB , CD = 2 cm. , CE = 8 cm. , $\overline{MD}$ is a tangent segment to the circle and MB = $\frac{1}{2}$ AB

Find the length of : $\overline{MD}$



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Unit 4

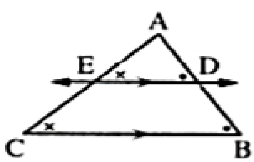
Lesson (1)

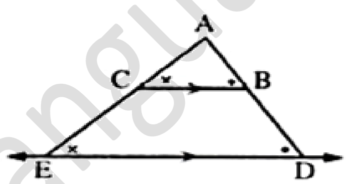
Parallel lines and proportional part

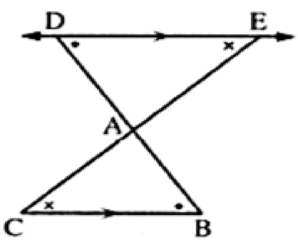
If a line is drawn parallel to one side of a triangle and intersects the other two sides or the lines containing them , then :

- The resulting triangle is similar to the original triangle
- It divides them into segments whose lengths are proportional

- **In each of the following figures :**







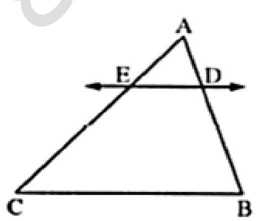
If $\overleftrightarrow{DE} \parallel \overline{BC}$ and intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ at D and E respectively , then :

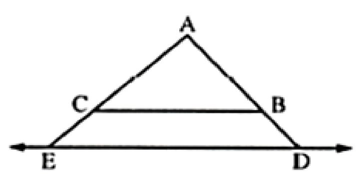
- $\Delta ADE \sim \Delta ABC$
- $\frac{AD}{DB} = \frac{AE}{EC}$ and from the properdies of the proportion , we get :

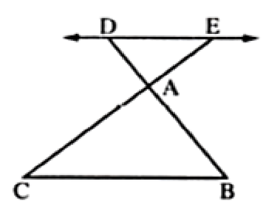
$\frac{AD}{AB} = \frac{AE}{AC}$, $\frac{AB}{DB} = \frac{AC}{CE}$

If a straight line intersects two sides of a triangle and divides them into segments whose lengths are proportional , then it is parallel to the third side of the triangle.

In each of the following figures :





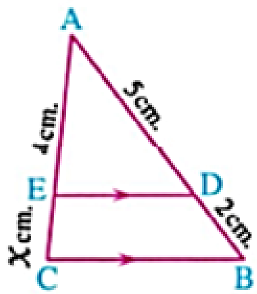


If $\frac{AD}{DB} = \frac{AE}{EC}$, then $\overleftrightarrow{DE} \parallel \overline{BC}$.

Example

In each of the following figures : $\overline{DE} \parallel \overline{BC}$ Find the value of x

1



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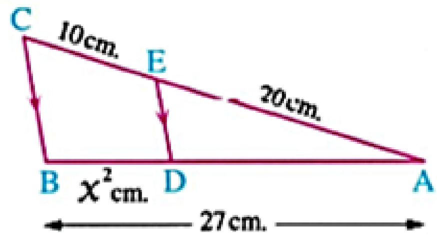

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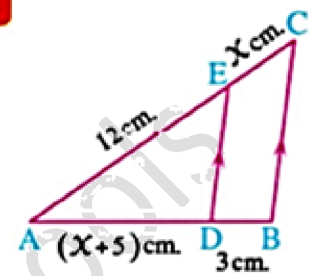

Solution……………………………………………………………………………………………

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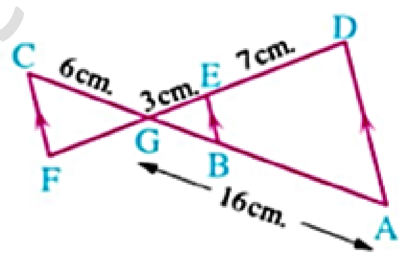
Example

In the opposite figure :

$\overline{AD} \parallel \overline{EB} \parallel \overline{FC}$, $\overline{AC} \cap \overline{DF} = \{G\}$, DE = 7 cm. , EG = 3 cm. , GC = 6 cm. , AG = 16 cm. Find the length of each of : $\overline{GF}$ and $\overline{GB}$



Solution……………………………………………………………………………………………

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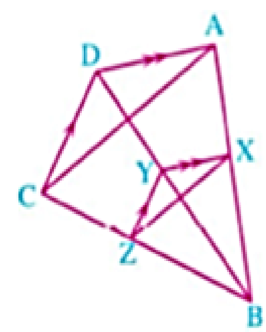
Example

In the opposite figure :

ABCD is a quadrilateral , $Y \in \overline{BD}$, $\overline{YX}$ is drawn such that $\overline{YX} \parallel \overline{DA}$ intersecting $\overline{AB}$ at X ,

$\overline{YZ}$ is drawn such that $\overline{YZ} \parallel \overline{DC}$ intersecting $\overline{BC}$ at Z

Prove that : $\overline{XZ} \parallel \overline{AC}$



Solution……………………………………………………………………………………………

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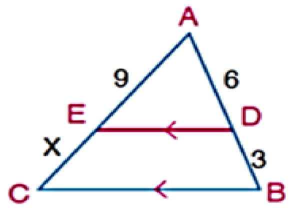
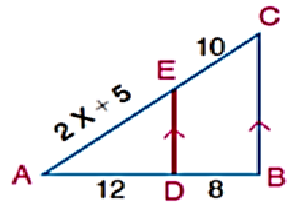
Exercises (1)

1 **In each of the following figures: $\overline{DE} \parallel \overline{BC}$. Find the numerical value of x** (length in centimetres).

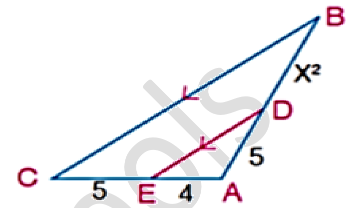
A



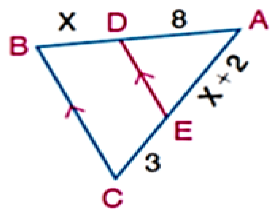
B



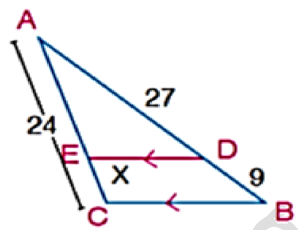
C



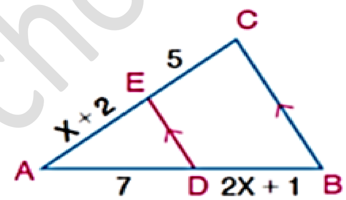
D



E



F



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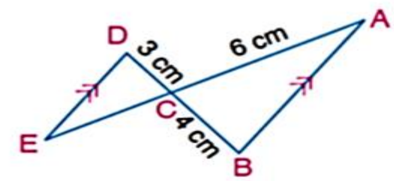
2 **In the figure opposite:** $\overline{AB} \parallel \overline{DE}$ and $\overline{AE} \cap \overline{BD} = \{C\}$

$AC = 6cm$, $BC = 4cm$ and $CD = 3cm$.

Find the length $\overline{AE}$



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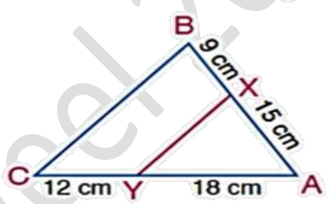
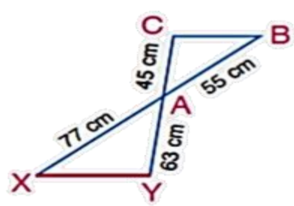
3 In each of the following figures, Is $\overline{XY} \parallel \overline{BC}$?

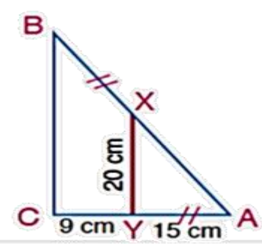
A



B



C



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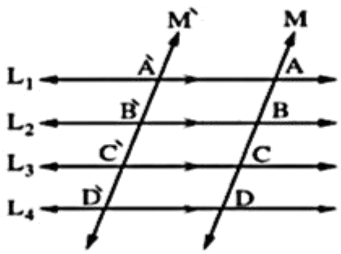
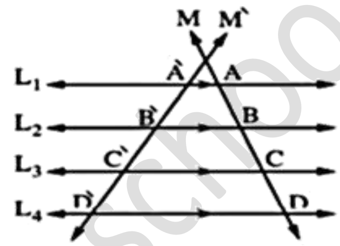
Lesson (2)

Talis' theorem

Theorem (2)

Given several coplanar parallel lines and two transversals , then the lengths of the corresponding segments on the transversals are proportional.





In the previous figures :

If $L_1 \parallel L_2 \parallel L_3 \parallel L_4$ and M , M' are two transversals

, then $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{AC}{A'C'}$

Remarks

If the lengths of the segments on the transversal are equal , then the lengths of the segments on any other transversal will be also equal.

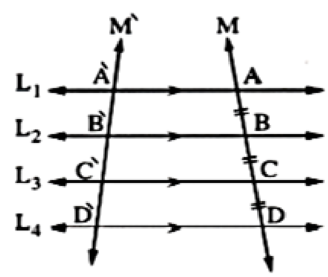
In the opposite figure :

If $L_1 \parallel L_2 \parallel L_3 \parallel L_4$,

M , M' are two transversals to them

and if AB = BC = CD

, then A'B' = B'C' = C'D'



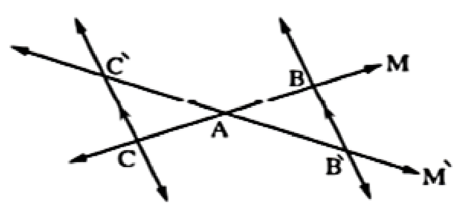
Special case

If the two lines M and M' intersect at

the point A and $\overleftrightarrow{BB'} \parallel \overleftrightarrow{CC'}$

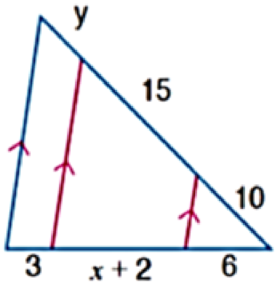
, then $\frac{AB}{AC} = \frac{AB'}{AC'}$

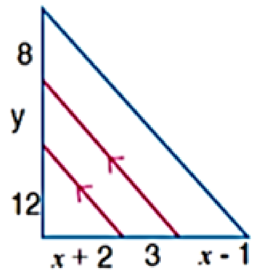
and conversely if $\frac{AB}{AC} = \frac{AB'}{AC'}$, then $\overleftrightarrow{BB'} \parallel \overleftrightarrow{CC'}$

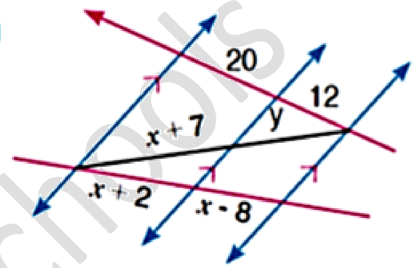


Exercises (2)

1 In each of the following figures, calculate the numerical values of x and y (lengths are measured in centimetres)

A


B


C


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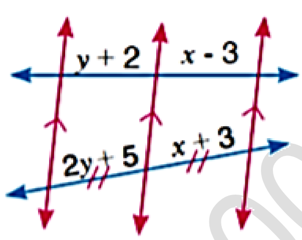
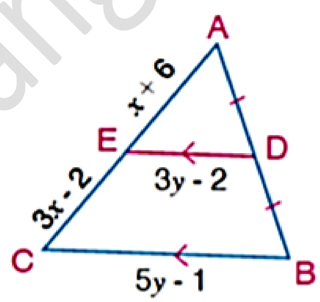
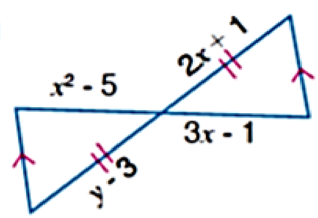
2 In each of the following figures, calculate the numerical values of x and y:

A


B


C


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3 $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$, $x \in \overline{AB}$, $y \in \overline{CD}$ and $\overline{XY} \,//\, \overline{BD} \,//\, \overline{AC}$

Prove that: $AX \times ED = CY \times EB$.

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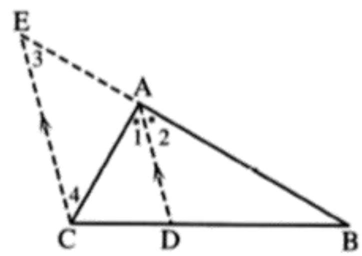
Lesson (3)

Angle bisector and proportional parts

Theorem (3)

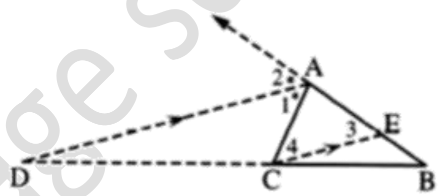
The bisector of the interior or exterior angle of a triangle at any vertex divides the opposite base of the triangle internally or externally into two parts , the ratio of their lengths is equal to the ratio of the lengths of the other two sides of the triangle.



$\because$ $\overrightarrow{AD}$ bisects $\angle$ BAC internally.

$\therefore$ $\boxed{\frac{BD}{DC} = \frac{AB}{AC}}$

, $\boxed{AD = \sqrt{AB \times AC - BD \times DC}}$



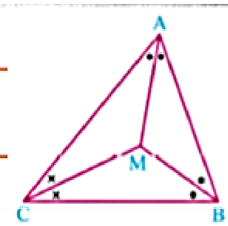
$\because$ $\overrightarrow{AD}$ bisects $\angle$ BAC externally.

$\therefore$ $\boxed{\frac{BD}{DC} = \frac{AB}{AC}}$

, $\boxed{AD = \sqrt{BD \times DC - AB \times AC}}$

Fact

The bisectors of angles of a triangle are concurrent.



Example

ABC is a triangle in which AB = 4 cm. , BC = 5 cm. , CA = 6 cm. , draw $\overrightarrow{AD}$ to bisect the angle A and intersects $\overline{BC}$ at D.

Find the length of each of : $\overline{BD}$, $\overline{DC}$, $\overline{AD}$

Solution……………………………………………………………………………

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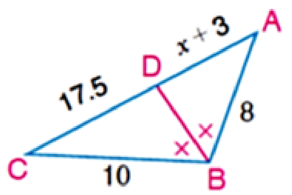
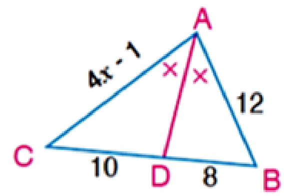
Exercises (3)

1 **In each of the following figures**: find the value of X (lengths are estimated in centimetres)

A



B



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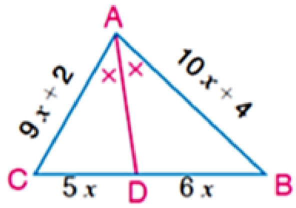
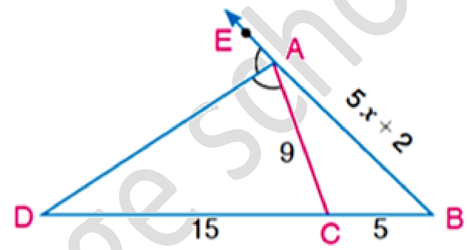
C



D



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2 **In the opposite figure :**

Prove that :

$\overrightarrow{BE}$ bisects $\angle ABC$

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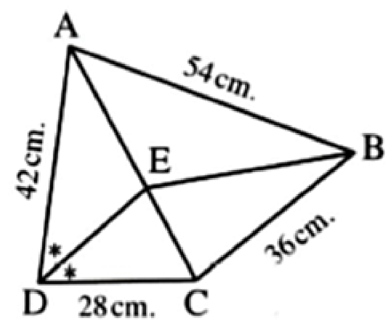

3 ABC is a triangle. its perimeter is 27cm. $\overrightarrow{BD}$ bisects $\angle$ B and intersects $\overline{AC}$ at D.

If AD = 4cm and CD = 5cm, find the length of $\overline{AB}$, $\overline{BC}$ and $\overline{AD}$

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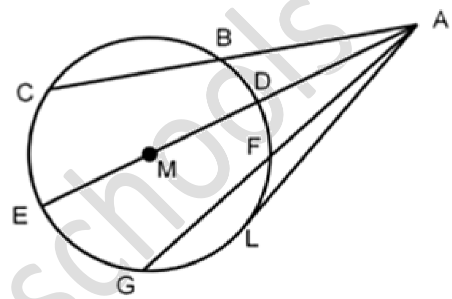
Lesson (4)

Power of a point with respect to a circle

We knew that

$AB \times AC = AF \times AG = AD \times AE = AL^2 = \text{constant}$



$AB \times AC = \text{constant}$

$AF \times AG = \text{constant}$

$AD \times AE = \text{constant}$

$AL^2 = \text{constant}$

So we called this constant "power of this point"
With respect to the circle M. we denote it by $P_M(A)$

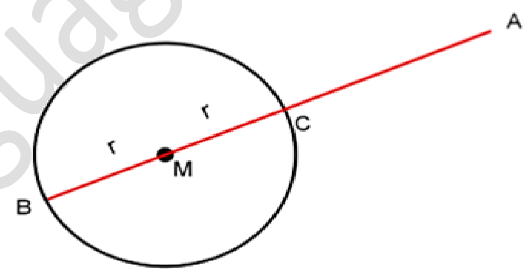
$\because\ AC = AM - r$

$\because\ AB = AM + r$

$\because\ P_M(A) = AC \times AB$

$\therefore\ \qquad = (AM - r)(AM + r)$

$\therefore\ \qquad = AM^2 - r^2$



Summarily we can prove it if the point A inside the circle or lies on the circle . It will be the same rule $P_M(A) = AM^2 - r^2$

Note (1)

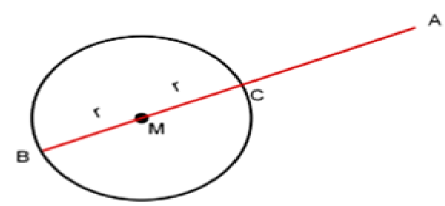
If point A outside the circle [$AM > r$]

$AM > r$ (squaring)

$(AM)^2 > r^2$

$(AM)^2 - r^2 > 0$

So, $P_M(A) > 0$ [positive value]



Note (2)

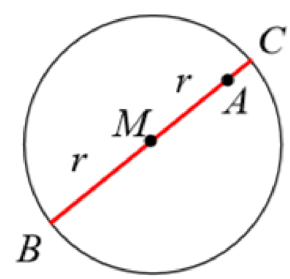
If point A inside the circle [$AM < r$]

$AM < r$ (squaring)

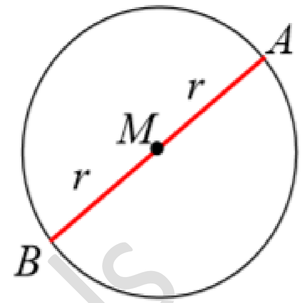
$(AM)^2 < r^2$

$(AM)^2 - r^2 < 0$

So, $P_M(A) < 0$ [negative value]



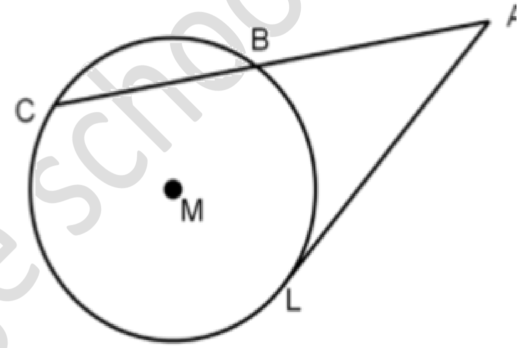
Note (3)



If point A on the circle [AM = r]

So, $P_M(A) = (AM)^2 - r^2 = 0$ [zero value]

Note (4)



If $P(A) = (AL)^2$

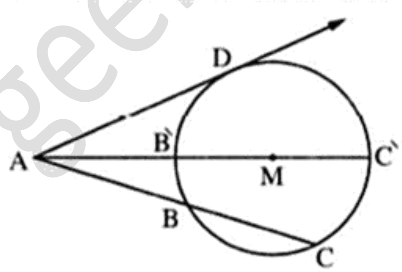
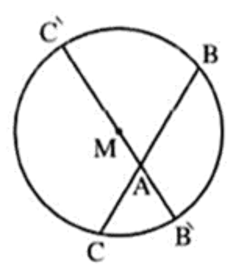
So, the length of the tangent drawn from A to circle M = $\sqrt{P(A)}$

Note (5)

The set of the point which have the same power with respect to distinct circles is called the principal axis of the two circles

If $P_M(A)=P_N(A)$,then A lies on the principal axis of two circles M and N

Summery

If A lies outside circle M , then :	**If A lies inside circle M , then :**
	
$P_M(A) = AB \times AC = AB' \times AC' = (AD)^2$	$P_M(A) = -AB \times AC = -AB' \times AC'$

$$P_M(A) = (AM)^2 - r^2$$

Exercises (4)

1 Find the power of the given point with respect to the circle M which its radius length is r:

A The point A where AM = 12cm and r = 9cm ……………………

B The point B where BM= 8 cm and r = 15 cm ……………………

C The point C where CM= 7 cm and r = 7 cm ……………………

D The point D where DM= $\sqrt{17}$ cm and r = 4 cm ……………………

2 If the distance between a point and the centre of a circle equals 25cm and the power of this point with respect to the circle equals 400. Find the radius length of this circle. ……………

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3 The radius length of circle M equals 20cm, A is a point distant 16cm from the centre of the circle, the chord $\overline{BC}$ is drawn where $A \in \overline{BC}$ and A B = 2 A C. Calculate the length of the chord $\overline{BC}$. ……………………

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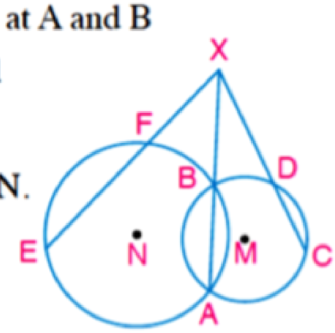
4 **In the figure opposite:** the two circles M and N are intersected at A and B where $\overrightarrow{AB} \cap \overrightarrow{CD} \cap \overrightarrow{EF} = \{X\}$, XD = 2 D C , EF = 10cm and $P_N(X) = 144$.



A Prove that $\overleftrightarrow{AB}$ is a principle axis to the two circles M and N.

B Find the length of $\overline{XC}$ and $\overline{XF}$

C Prove that CDFE is a cyclic quadrilateral.

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